Child Labor as an Operational Lever: Heterogeneous Effects of Interventions in Cocoa Supply Chains

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Abstract. Problem definition: More than 1.5 million children engage in child labor in cocoa production in Côte d'Ivoire and Ghana despite decades of interventions. Most of this work occurs within smallholder farm households, where child labor is one of the levers adults use to balance tradeoffs among production, consumption, and finances. These tradeoffs are household-specific, creating a design problem for responsible-sourcing programs: an intervention that reduces child labor in some households may increase it in others.

Methodology/results: We develop a model of a cocoa household that chooses its consumption, borrowing, saving, and child labor use under harvest uncertainty, and use it to predict how common interventions affect child labor. We then test the predictions using survey data collected by an NGO partner in Ghana. Our model shows that interventions affect child labor through three forces: resource relief, labor-productivity effects, and consumption expansion. Cash transfers and improved savings access operate primarily through resource relief and strictly reduce child labor when the household can borrow or save; otherwise, child labor is unchanged. Improved credit access can either lower or raise child labor depending on whether repayment relief dominates debt-financed consumption. Production support can reduce or increase child labor depending on whether resource relief dominates the labor-productivity and consumption-feedback forces. Under multiplicative production risk, labor-substituting production support weakly reduces child labor for all households, whereas labor-complementary support can either strictly lower or strictly increase child labor. All aforementioned interventions raise household welfare but have nuanced effects on consumption. The survey evidence is consistent with our framework, though descriptive rather than causal: cash transfers show no systematic increase in child labor, while loan repayment ability and production-related benefits are associated with lower child labor among more constrained households and higher child labor among less constrained or higher-capacity households.

Managerial implications: Responsible-sourcing programs should evaluate and target interventions based on household characteristics. Important household characteristics, such as income, spending, loan repayment capacity, or production scale, are already measured in standard surveys and can help predict when a given intervention might decrease child labor or backfire.

Key words: child labor; intervention design; household operations; heterogeneous treatment effects; responsible sourcing; development operations; cocoa supply chains

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1. Introduction

Child labor remains pervasive in the cocoa supply chains of Côte d'Ivoire and Ghana and, despite more than two decades of awareness and coordinated action, more than 1.5 million children are estimated to engage in often hazardous work in cocoa production (Tulane University 2015, NORC 2020, ICI 2020, Foubert 2021). Much of this labor occurs within smallholder farm households, where child labor functions as an operating lever that adults use to manage production needs, consumption demands, and financial constraints over an uncertain harvest season (Nkamleu and Kielland 2006, Vigneri et al. 2016, ICI 2020). Yet operations management research has paid limited attention to the household-level tradeoffs that drive the use of child labor, and this omission matters for responsible sourcing programs. Because these tradeoffs vary across households, the same intervention can reduce child labor in some while inadvertently increasing it in others.

To fill this gap, this paper introduces a framework for identifying when responsible-sourcing interventions may backfire and increase child labor. The framework relies on a parsimonious model that formalizes the tradeoffs facing households during a single cocoa production season. We consider a household that enters the season with an initial cash position and chooses current consumption, how much to borrow or save, and how much child labor to use on the farm, under a stochastic harvest cashflow. Child labor creates disutility, but it increases harvest cashflow and therefore strengthens the end-of-season financial position. A key feature of the model is that borrowing and saving carry different rates: the household borrows at a higher rate than it earns on savings. This realistic wedge allows the model to distinguish borrowing, saving, and financially neutral households, and to separate the effects of improved credit access from those of improved savings access. Despite its simplicity, the model is quite general: it imposes mild structure on consumption utility, child-labor disutility, cash future value, and harvest production.

We then analyze several common responsible sourcing interventions, which we model as shifts in one of four household model parameters. Cash transfers raise the household's cash position; improved credit access lowers the cost of borrowing; improved savings access raises the return to saving; and production support improves the farm's harvest cashflow. (We use "production support" as shorthand for subsidized inputs, agronomic training, help acquiring hired labor or equipment, price premiums, or other production- or revenue-enhancing benefits.)

The comparative statics show that the household's cash position is central to intervention effectiveness because it determines whether the household borrows, saves, or is financially neutral. Across these regimes, interventions affect child labor through three forces. First, resource relief improves the household's end-of-season financial position and lowers the marginal value of additional harvest cashflow. This force pushes child labor down when child labor is useful at the margin for generating harvest cashflow; when that margin is locally absent, child labor is unchanged. Second, labor-productivity effects change the marginal return to child labor in production. They push child labor up when the intervention makes child labor more productive, and push it down when the intervention substitutes away from child labor. Third, consumption expansion can work against resource relief: when an intervention supports higher current consumption that must be financed against the harvest, the value of harvest cashflow rises and child labor can become more attractive.

These forces imply different backfire risks across common responsible-sourcing interventions. Cash transfers increase the household's cash position without creating repayment obligations or raising the marginal product of child labor. They therefore weakly reduce child labor or leave it unchanged. The reduction is strict only when part of the transfer improves the household's net pre-harvest position and child labor is valuable at the margin for harvest cashflow. If the household is financially neutral, the transfer is absorbed one-for-one into current consumption and child labor does not change. Improved savings access works similarly for households that save: it raises the return on an existing buffer and creates no new repayment obligation, so it weakly lowers child labor, with a strict reduction when the same marginal harvest-cashflow force is present.

Improved credit access is different because it changes the cost of borrowing. For borrowers with large existing repayment exposure, a lower borrowing rate mainly reduces the cost of debt the household was already going to carry; harvest cashflow becomes less valuable at the margin, and child labor falls. For borrowers with smaller repayment exposure, the same improvement can instead support additional current consumption financed by new borrowing; that larger repayment obligation can raise the value of harvest cashflow and increase child labor. Production support is also different because it directly changes the production environment: it strictly raises household welfare under our output-improvement assumption, but its child-labor effect depends on whether harvest-resource relief dominates the labor-productivity and consumption-feedback forces. In a simpler benchmark where uncertainty impacts harvest cashflow multiplicatively, production support that does not increase the marginal product of child labor weakly reduces child labor for every cash position; in contrast, labor-complementary support can reduce child labor for some cash positions and increase it for others.

We then examine whether these patterns appear in surveys run by our NGO partner of cocoa-producing households in Ghana. The survey variables are not obtained through randomized experiments and are not perfectly aligned with our model's primitives, so we construct suitable proxies and interpret the estimates as adjusted associations rather than causal effects. Participation in a cash-transfer program is the closest empirical counterpart to a positive cash-position shift. The household's self-reported ability to repay its outstanding loan is an (admittedly imperfect) proxy for access to a household's access to cheaper credit. And production-related benefits are the closest proxy for improvements in the farm production environment. Output variables record whether the household used any child labor during the past 7 days, the past 6 months, or the past year. We compute heterogeneous effects of the interventions by proxying for a household's cash position through its income, revenue, or spending.

Three empirical patterns are visible. Our cash-transfer proxy has estimated associations with child labor that are generally close to zero or negative across income and spending groups. In some specifications, the reductions are larger among households with higher income or spending, which is consistent with the model's prediction that cash transfers reduce child labor only when they improve the net pre-harvest position rather than being fully absorbed into current consumption. Importantly, cash-recipient status does not show the increase among higher-income or higher-spending households that would indicate expansion risk. Loan-repayment ability shows a different pattern, especially across spending groups: associations are negative or near zero for households with lower spending and more positive for households with higher spending. Production-related benefits show the clearest expansion pattern: associations are negative or near zero for households with lower revenue or net income and positive for households with higher revenue or net income. These patterns are descriptive and imprecise, but they match the model's main design message: assigning a unique "success" label to an intervention is insufficient, and effectiveness depends on the household's cash position, borrowing exposure, and production capacity.

As a result, our main contribution is a backfire-aware framework for responsible-sourcing intervention design in settings where the targeted harmful practice is an endogenous operating lever. Our prescription is that to mitigate backfire risk, responsible-sourcing programs should target interventions based on household characteristics rather than average intervention effects alone. A program can, for example, reduce the risk of backfire by matching credit and production support to households whose characteristics suggest the resource-relief force is likely to dominate, and by relying on cash and savings support when broad reach and household characteristics that might lead to backfire risk are hard to measure.

The remainder of the paper is organized as follows. Section 2 situates the contribution within prior work in economics, responsible sourcing, and smallholder operations. Section 3 develops the household model. Section 4 analyzes the comparative statics of various interventions. Section 5 describes the ICI survey data from Ghana, the intervention definitions, and the empirical strategy. Section 6 presents the empirical patterns and relates them to the model.

2. Related Literature

Our work relates to three research streams. The first models child labor as a household response to poverty, missing markets, and intertemporal constraints. The second estimates how cash, credit, and agricultural interventions affect child labor in low-income settings. The third stream studies responsible sourcing and smallholder operations. Our focus is the smallholder cocoa household as an operating unit: interventions shift different household primitives, and child labor responds according to the household's wealth–debt position, productive capacity, and harvest uncertainty.

Child Labor, Credit Constraints, and the Poverty Threshold

The classical analytical literature studies child labor as a household response to poverty, missing markets, and intertemporal constraints. Basu and Van (1998) and Basu (1999) formalize the subsistence logic behind child labor, while Ranjan (2001) shows how initial household resources and child wages shape this decision and Basu et al. (2010) show that functioning labor markets can reduce child labor in a dynamic model with adult savings and bequests to children. A related set of papers focuses more directly on credit constraints: Baland and Robinson (2000) show that, when households cannot borrow against future income, child labor can exceed the socially efficient level; Jafarey and Lahiri (2002) compare local and perfect international credit markets and their implications for child labor. Other work studies legal and schooling-based interventions, including child labor bans, penalties, and mandatory schooling (Ranjan 1999, Doepke and Zilibotti 2005, Maffei et al. 2006, Dessy and Knowles 2008, Tsuyuhara et al. 2014). These papers provide the household-level primitives for our model, but they do not examine financial and operational interventions in a stochastic farm production setting.

Our focus on agricultural households means that child labor serves simultaneously as a risky production input and a financing mechanism. This dual role generates three forces through which any intervention affects child labor – resource relief, labor-productivity, and consumption expansion – whose relative magnitudes depend on the household's financial and production characteristics. We show why the same intervention can reduce child labor in some households and increase it in others, and identify the observable household characteristics that determine which force dominates.

Cash Transfers, Credit Access, and Agricultural Inputs

A large empirical literature evaluates unconditional cash transfers in low-income settings (Robertson et al. 2013, Haushofer and Shapiro 2016, Bastagli et al. 2016), with a smaller subset focused on child labor. Using evidence from South Africa's Old Age Pension Program and the Lesotho Child Grants Programme, respectively, Edmonds (2006) and Sebastian et al. (2019) show that unconditional cash transfers reduce children's labor and improve their schooling outcomes, with heterogeneous effects across gender and household types. Chong and Yáñez-Pagans (2019) complicate this picture: Bolivia's old-age pension increases child labor among boys in rural households.

A parallel empirical literature studies microcredit and child labor. Islam and Choe (2013), Edmonds and Theoharides (2020), and Hossain (2023) use randomized trials in Bangladesh and the Philippines and find that productive microcredit can raise child labor; Hazarika and Sarangi (2008) reache the same conclusion in rural Malawi.

Productivity programs show a similar pattern. Evidence from Malawi suggests that input subsidies can raise both the incidence and the intensity of child labor when farm productivity rises (Frempong 2023), and that these effects depend on household institutions: Kamanga and Mwale (2026) find that subsidies raise child labor in matrilineal households but not in patrilineal ones. Related evidence from India reports that mechanical technologies reduce child labor among Indian farmers above the poverty line but not below it, while biochemical inputs have mixed effects on child labor (Self and Grabowski 2009). Finally, Luckstead et al. (2019) calibrate a smallholder farm model and estimate that a 2.81% cocoa price premium would eliminate the worst forms of child labor.

Our paper contributes to these empirical streams by offering a model that can rationalize the mixed findings in this literature through three forces – resource relief, labor-productivity complementarity, and consumption expansion. While demographic and institutional factors clearly matter, our framework shows that the three forces above generate heterogeneous responses to interventions through financial and production conditions that cut across demographic categories. Our data, while not sufficient to establish causal effects, produce adjusted associations that are consistent with this mechanism, suggesting that the heterogeneity documented across settings may reflect a common operational logic that complements demographic and institutional explanations.

Responsible Sourcing and Smallholder Operations

The operations-management literature on sustainable and responsible sourcing has expanded over the last decade (Chen and Lee 2017, Kalkanci and Plambeck 2020, Ramchandani et al. 2025, Ha et al. 2023). Within this stream, Cho et al. (2019) study auditing and pricing by a firm seeking to reduce child labor in its supply chain. Related operations work also shows that wealth and liquidity constraints shape operational strategy in low-income markets, including distribution, financing, and marketing decisions (Calmon et al. 2022).

A related stream studies smallholder operations directly. The development economics literature on smallholder farming is reviewed in Angelsen (1999), and the operations management community has devoted increasing attention to the topic (An et al. 2015, Liao et al. 2019). Chintapalli and Tang (2022) and Park et al. (2026) compare financing strategies a government can use to support smallholder farmers; de Zegher et al. (2019), de Zegher et al. (2018), Warnes et al. (2025), and Yi et al. (2021) analyze market-based solutions; Tang et al. (2024) contrast input and output subsidies; and Agrawal and Zhang (2024) compare flexible and fixed premiums for certified commodities.

Our paper builds on the microeconomic frictions in this literature – stochastic yields and liquidity constraints – but reframes the design question. Rather than asking how a firm or government should set contract terms or subsidies to maximize farmer welfare, we ask how those instruments reshape household operating tradeoffs, and whether that reshaping can backfire. In our model, child labor is an endogenous operational lever the household adjusts in response to any shift in its financial position, production capacity, or consumption margin. Cash transfers and savings support weakly reduce child labor, while credit access and production support can increase it – depending on the household's financial regime and production characteristics. This heterogeneity is central to intervention design but absent from models focused on average household responses.

3. Model

This section develops a model of a farm household that uses child labor as an operational lever while choosing current consumption and how much to borrow or save. The model serves two purposes. First, it allows us to characterize how interventions such as cash transfers, credit access, savings access, and production or revenue support change the tradeoffs faced by the household, and when those changes can create pressure to use more child labor rather than less. Second, it provides a framework for interpreting the empirical patterns documented in §6.

The model is deliberately parsimonious, but general. It focuses on household decisions and states that can be inferred from farmer surveys, while imposing minimal structure on household utility and harvest production. We introduce the household problem in §3.1 and derive preliminary results in §3.2. §4 then analyzes the intervention comparative statics.

3.1. Household Model

A household enters a new growing season with an initial cash position of x cash units. The household makes four key decisions: its consumption during the season c, its new borrowing b, its new savings s, and how much child labor to use on the farm ℓ. For analytical simplicity and consistent with literature in operations management and economics (Baland and Robinson 2000, de Zegher et al. 2018), we assume that all decisions are made simultaneously at the start of the season.

Consumption during the season c yields utility $u(c)$. We consider u to be an extended-real function, $u : \mathbb{R} \to \mathbb{R} \cup \{-\infty\}$, so that its effective domain can model other consumption constraints, e.g., a lower consumption bound for subsistence needs. On its effective domain, u is twice differentiable, strictly increasing ($u' > 0$), and strictly concave ($u'' < 0$).

The household can increase its consumption by borrowing, and can save excess cash. Specifically, the household can borrow an amount $b \geq 0$ at gross borrowing rate $r_b > 1$ (implying interest rate $r_b - 1$), repaying the full amount $b \cdot r_b$ at the end of the season. For simplicity, we do not consider credit limits or increasing marginal borrowing costs; including them would not change our insights. The household can also save an amount $s \geq 0$ at gross savings rate $r_s \in [1, r_b)$ (lower than the borrowing rate), receiving the principal plus interest $s \cdot r_s$ at the end of the season. The household finances its consumption and savings from its initial liquidity position and its borrowing, so its decisions must satisfy the constraint $x + b = c + s$.

Using child labor ℓ carries non-financial disutility $d(\ell)$, reflecting both direct costs of removing children from school or leisure and any social stigma associated with the practice (Basu et al. 2010, Maffei et al. 2006). We take d as an extended-real function, $d : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$, so that its effective domain can model constraints on ℓ due to, e.g., the availability of child labor in the household. On its effective domain, d is twice differentiable, strictly increasing ($d' > 0$), and strictly convex ($d'' > 0$), consistent with increased marginal costs at higher levels of child labor.

After choosing (c, b, s, ℓ), the household's cashflow from the new harvest is $\Phi(\ell, \theta, \xi)$. Here, θ is a production index that models the household's level of agronomic technology or use of productive inputs, and ξ is an exogenous random shock modeling variation in production yields or in the selling price of outputs or costs of inputs. For analytical convenience, we assume that all expectations of relevant quantities are finite, and that interchanging differentiation and expectation is valid. The harvest cashflow $\Phi(\ell, \theta, \xi)$ is second-order differentiable and is increasing and concave in ℓ for each (θ, ξ), i.e., $\Phi_\ell(\ell, \theta, \xi) \geq 0, \Phi_{\ell\ell}(\ell, \theta, \xi) \leq 0$, reflecting that child labor increases harvest cashflow, but with diminishing marginal returns. We model interventions such as offering the household subsidized inputs, agronomic training, price premiums, or other forms of production or revenue improvements (in short, "production support") through an increase in the production index θ, and assume that $\Phi_\theta(\ell,\theta,\xi) \geq 0$ almost surely and $\mathbb{P}[\Phi_\theta(\ell,\theta,\xi) > 0] > 0$ for all relevant (ℓ,θ).

The household's end-of-season financial position is[1] $\Phi(\ell,\theta,\xi) + r_s s - r_b b$, and is valued through a continuation function v. For simplicity, we assume that v embeds any discounting of future value. We take $v : \mathbb{R} \to \mathbb{R}$ as a twice differentiable function that is strictly increasing ($v' > 0$) and concave ($v'' \leq 0$), reflecting risk aversion and diminishing returns from an increased financial position.

The household chooses its consumption c, borrowing b, saving s, and child labor use ℓ to maximize its net welfare, which entails solving the problem:

$$\begin{aligned} &\underset{c,b,s,\ell}{\text{maximize}}\ u(c) - d(\ell) + \mathbb{E}\Big[v\big(\Phi(\ell,\theta,\xi) + r_s \cdot s - r_b \cdot b\big)\Big] \\ &\text{subject to } c + s = x + b, \quad s \geq 0, \quad b \geq 0. \end{aligned} \tag{1}$$

We keep our model minimal and focus on primitives and decisions that help rationalize the empirical observations in §6. Notably, the interventions discussed in §6 map to primitives in the household's problem (1): cash transfers or in-kind support increase the liquidity position x, credit access lowers r_b, and production or revenue support increases θ. We include a saving decision with a different rate $r_s \neq r_b$ for realism and generality, and to help isolate the benefits of improved credit access by changing r_b. Our model deliberately omits other important decisions that a household may make during the production season. For instance, we do not model investments in productive inputs; §EC.3.2 of the Appendix extends the cash-transfer results to allow endogenous input purchases and shows that the qualitative insights survive, with complementarity between the purchased input and child labor emerging as an additional – but not new – determinant of the child-labor response.

3.2. Preliminary Results

The household's problem highlights a joint consumption-production tradeoff. Current consumption is valuable and its marginal value can be especially high when the household lacks ample cash x, because consumption is constrained by x and u is increasing and concave. But higher consumption can be financed: the household either saves less or borrows more, both of which increase its ability to consume, while also reducing its end-of-season financial position. Child labor can relax this intertemporal friction. Although an additional unit of child labor is costly through $d(\ell)$, it raises harvest cashflow and increases the household's end-of-season financial position. Child labor thus

[1] The end-of-season financial position is $x + b - c - s + \Phi(\ell,\theta,\xi) + r_s \cdot s - r_b \cdot b$, which equals the stated form because $x + b = c + s$.

acts both as a production input and as a financing mechanism: by increasing potential harvest cashflow, it lowers the costs associated with a higher consumption, lower savings, or larger debt.

We first introduce a supporting result that will streamline our subsequent analysis. Let (ℓ^*, c^*, s^*, b^*) denote an optimal solution of (1). Our first result shows that the household will not simultaneously save and borrow, thereby simplifying the household's problem.

LEMMA 1 **(No simultaneous borrowing and saving).** *For any optimal solution in problem* (1),

$$b^* = (c^* - x)^+, \qquad s^* = (x - c^*)^+,$$

where $w^+ := \max(w, 0)$. *In particular, the household does not simultaneously borrow and save.*

Proofs for all results are in the appendix.

The intuition behind Lemma 1 is that borrowing is used only to finance consumption rather than savings because the borrowing rate is strictly larger than the savings rate ($r_b > r_s$).[2]

Lemma 1 reduces the household's problem to a choice of (ℓ, c) in three possible financial regimes. In the *saving regime*, $c < x$ and the household does not borrow but saves $s^* = x - c$, so the relevant gross rate is $r = r_s$. In the *borrowing regime*, $c > x$ and the household optimally borrows $b^* = c - x$ and does not save, so the relevant gross rate is $r = r_b$. Lastly, in the *financially neutral* regime, $c = x$ so the household consumes its entire initial cash and neither borrows nor saves, $b^* = s^* = 0$.

The Lemma also allows simplifying the household's problem. For any (ℓ, c), we can rewrite the household's (optimal) end-of-season financial position and its objective, respectively, as:

$$\bar{Y}(\ell, c; x, \theta, \xi) := \Phi(\ell, \theta, \xi) + r_s (x - c)^+ - r_b (c - x)^+ \tag{2a}$$

$$\bar{V}(\ell, c; x, \theta) := u(c) - d(\ell) + \mathbb{E}\left[v\left(\bar{Y}(\ell, c; x, \theta, \xi)\right)\right]. \tag{2b}$$

With this notation, the household's problem can be concisely summarized as:

$$\max_{c,\ell} \bar{V}(\ell, c; x, \theta). \tag{3}$$

A star "*" denotes quantities evaluated at the optimal solution, and we use the shorthand notation $\Phi^* := \Phi(\ell^*, \theta, \xi)$, $\bar{Y}^* := \bar{Y}(\ell^*, c^*; x, \theta, \xi)$, $\bar{V}^* := \bar{V}(\ell^*, c^*; x, \theta)$ and an analogous notation for the derivatives of these quantities (when derivatives exist). We omit showing all arguments of the functions when no confusion can arise, and we write $\ell^*(\eta), c^*(\eta), \bar{Y}^*(\eta), \bar{V}^*(\eta)$ when seeking to highlight the dependency on a specific subset of parameters $\eta \subseteq \{x, r_b, r_s, \theta\}$.

Subsequently, we primarily consider interior optimal solutions for problem (3), but we also discuss some results that arise when considering boundary conditions on consumption or child

[2] For the same reason, Lemma 1 and the equality $c^* + s^* = x + b^*$ would optimally hold even if our base model had required $b + x \geq c + s$ rather than requiring equality. We adopted the equality constraint primarily for parsimoniousness.

labor. Our standing assumptions imply that the objective $\bar{V}$ is strictly jointly concave in (ℓ, c) in the interior of the domain $\text{int}(\text{dom}(d)) \times \text{int}(\text{dom}(u))$. However, $\bar{V}$ is only differentiable (in fact, twice differentiable) for $c \neq x$ (see Lemma EC.1 in the Appendix). Therefore, the optimal interior solution to the household's problem is uniquely characterized by the following first-order conditions:

$$d'(\ell^*) = \mathbb{E}\left[v'(\bar{Y}^*)\Phi_\ell(\ell^*, \theta, \xi)\right] \tag{FOC-ℓ}$$

$$\exists\, \rho^* \in \mathcal{R}(c^*, x) \;\; : \;\; u'(c^*) = \rho^* \mathbb{E}\left[v'(\bar{Y}^*)\right], \tag{FOC-c}$$

where ρ^* denotes the relevant rate depending on the household's financial regime (savings, financially neutral, or borrowing), and $R(c;x)$ is the set of relevant interest rates:

$$\mathcal{R}(c;x) := \begin{cases} \{r_s\}, & c < x, \\ [r_s, r_b], & c = x, \\ \{r_b\}, & c > x. \end{cases}$$

At an optimal interior solution, the household balances several margins simultaneously. The labor choice equates the marginal disutility of child labor with the expected continuation-value gain from the additional harvest resources it produces. The consumption choice equates the marginal utility of current consumption with the continuation-value cost of drawing resources from the end-of-season position. When the household expands current consumption, especially from a low-x state where $u'(c)$ is high, it places pressure on the end-of-season budget. That pressure raises the marginal value of harvest cashflow and, with it, the value of child labor. When an intervention instead eases the household's net end-of-season budget pressure without raising the return to farm production, the marginal value of harvest cashflow falls, and with it the household's reliance on child labor for consumption smoothing or repayment.

Subsequently, let $q^*(x) := x - c^*(x)$ be the household's net pre-harvest financial position after current consumption. Thus $q^*(x) < 0$ is the borrowing regime, $q^*(x) > 0$ is the saving regime, and $q^*(x) = 0$ is the financially neutral regime.

For our subsequent comparative statics, we note that within the borrowing or saving regime with active rate $\rho^* \in \{r_b, r_s\}$, respectively, we have:

$$\bar{V}^*_{\ell c} = \rho^* A^*_\ell \geq 0, \qquad \bar{V}^*_{\ell x} = -\rho^* A^*_\ell \leq 0, \qquad \text{where} \quad A^*_\ell := \mathbb{E}[-v''(\bar{Y}^*)\Phi^*_\ell] \geq 0. \tag{4}$$

In our model, consumption and labor are complements (as higher current consumption c lowers the pre-harvest financial position $q = x - c$, raises the marginal value of harvest cashflow, and makes child labor more valuable), but labor and cash are substitutes (as a higher cash position, holding consumption fixed, improves the pre-harvest financial position and makes child labor less valuable). But the strict complementarity and substitutability are determined by A_ℓ^*, which measures how sensitive the marginal value of child labor is to changes in the household's terminal financial position. When $A_\ell^* = 0$, the marginal value of child labor does not change with changes in terminal financial position, which shuts down an important channel through which certain (financial) interventions can impact child labor. $A_\ell^* = 0$ can occur if the continuation value v is locally linear in terminal resources, or if child labor has no marginal value in states where the continuation-value curvature matters.

4. Comparative Statics of Child Labor Interventions

We now use the analytical model to characterize the effect of different interventions on the household's child labor use, and to understand how this depends on the household's financial regime.

4.1. Cash Transfers

We first discuss how cash transfers – modeled as an increase in the household's initial cash position x – impact the household's use of child labor and its consumption and welfare.

Proposition 1 **(Cash transfers).** *Consider the range of values for the household's cash position x for which an interior optimal solution exists. Then:*

(i) There exist thresholds $x_1 \le x_2$ (possibly $\pm\infty$) such that the household is in the borrowing regime for $x < x_1$, in the financially neutral regime for $x_1 < x < x_2$, and in the saving regime for $x > x_2$.

(ii) Child labor $\ell^*(x)$ is decreasing in x in the borrowing and in the saving regime (strictly if and only if labor and cash are locally strict substitutes, $A_\ell^* > 0$), and is unchanged in the financially neutral regime. More precisely,

$$\begin{aligned} &\text{if } x < x_1 \text{ or } x > x_2: \quad && \tfrac{\partial \ell^*(x)}{\partial x} \le 0 \quad \text{and} \quad \tfrac{\partial \ell^*(x)}{\partial x} < 0 \iff A_\ell^* > 0 \\ &\text{if } x_1 < x < x_2 \quad : && \tfrac{\partial \ell^*(x)}{\partial x} = 0. \end{aligned}$$

(iii) Current consumption $c^*(x)$ is increasing in x in the borrowing regime and in the saving regime, and is strictly increasing in the financially neutral regime. More precisely:

$$0 \le \frac{\partial c^*(x)}{\partial x} < 1 \text{ if } x < x_1 \text{ or } x > x_2 \qquad \text{and} \qquad c^*(x) = x \text{ if } x_1 < x < x_2.$$

(iv) Household welfare $\bar{V}^*(x)$ is strictly increasing in x.

A cash transfer increases the household's cash position x without changing the marginal product of child labor. In the borrowing and saving regimes, current consumption rises by less than one-for-one with x, so the household's net pre-harvest financial position $q^*(x) = x - c^*(x)$ improves. In the borrowing regime this means less debt pressure carried into the harvest; in the saving regime, it means a larger financial buffer. In both cases, the marginal continuation value of harvest cashflow falls, and therefore the marginal benefit of child labor falls. The strength of this child-labor response is governed by A_ℓ^*. When cash and child labor are strict substitutes ($A_\ell^* > 0$), child labor strictly decreases with the increase in the cash/terminal resource position. If $A_\ell^* = 0$, the marginal value of child labor is locally insensitive to changes in terminal resource position, so the cash transfer has no first-order effect on child labor.

The financially neutral regime is different. There, $c^*(x) = x$, so the transfer is absorbed one-for-one into current consumption and $q^*(x) = 0$ remains unchanged. Terminal resources and the labor first-order condition are therefore unchanged, and child labor is unaffected by the intervention. Thus cash transfers can raise consumption and welfare while leaving child labor unchanged for households in the financially neutral regime, but they weakly reduce child labor whenever part of the transfer improves the household's net pre-harvest financial position.

Several extensions in Appendix §EC.3.1 prove that these insights are robust. Proposition EC.1 considers a household with a minimum consumption level $c \geq c_s + \alpha x$, where c_s is a fixed subsistence consumption level and $\alpha \in [0, 1)$ is the fraction of the household's cash devoted to consumption. The proposition shows that if the consumption constraint is binding, cash transfers will reduce the household's child labor use (strictly under a condition similar to Proposition 1), will increase its consumption (strictly if $\alpha > 0$), and will strictly increase the household's welfare. Proposition EC.2 considers a separate setting where the child labor decision is constrained due to availability of child labor, $\ell \leq \bar{\ell}$. If this labor constraint is binding, cash transfers will strictly increase the household's consumption and welfare, and will leave child labor use unchanged ($\ell^* = \bar{\ell}$), but would, however, lower the shadow price of the child labor constraint. For both results, when cash transfers are sufficiently large to make the respective constraint non-binding, the household will find itself in an unconstrained regime and the results of Proposition 1 would then apply.

Lastly, Appendix §EC.3.2 considers the setting when the household can invest some of its cash position to improve its farm production. This makes a cash transfer intervention qualitatively similar to a production support intervention (discussed in §4.4), because it creates incentives for the household to expand its use of child labor when this is complementary to the farm investment. However, §4.4 and §EC.3.2 of the Appendix show that when the investment is not a strong complement to child labor, the qualitative effects of Proposition 1 hold and cash transfers reduce the household's use of child labor, while increasing its consumption and welfare.

4.2. Credit Access

We next examine how improved credit access, modeled as a decrease in r_b, affects child labor. Improved credit access can operate through two opposing effects: it can reduce the debt burden and repayment pressure, but it can also encourage additional current consumption financed by new borrowing. The sign of the child-labor response will thus depend on which effect dominates. On the consumption side, the relevant determinant for the effect will be the household's local absolute risk tolerance for consumption,

$$T_u(c) := \frac{u'(c)}{-u''(c)}. \tag{5}$$

This is the reciprocal of the coefficient of absolute risk aversion, and can be equivalently defined as: $T_u(c) = -\left(\frac{d \log u'(c)}{dc}\right)^{-1}$. Thus $T_u(c)$ measures how quickly marginal utility changes around c. A smaller $T_u(c)$ means that marginal utility changes sharply with small changes in consumption, so the household strongly resists a decrease in consumption. A larger tolerance $T_u(c)$ means that marginal utility is locally flatter, so larger absolute changes in consumption are needed to generate the same change in marginal utility, so the household can tolerate a larger decrease in consumption.

PROPOSITION 2 **(Credit access).** *Consider a range of values of r_b for which an interior optimal solution exists and the household is in the borrowing regime.*

(i) If labor and consumption are locally strict complements ($A^*_\ell > 0$), child labor is (strictly) increasing in r_b if and only if the borrowed amount is (strictly) larger than the risk tolerance,

$$\frac{\partial \ell^*}{\partial r_b} \geq 0 \quad \Longleftrightarrow \quad b^* = c^* - x \geq T_u(c^*). \tag{6}$$

Otherwise ($A^*_\ell = 0$), child labor is unaffected by a change in r_b, $\frac{\partial \ell^*}{\partial r_b} = 0$.

(ii) Current consumption $c^*(r_b)$ is strictly decreasing in r_b.

(iii) Household welfare $\bar{V}^*(r_b)$ is strictly decreasing in r_b.

Proposition 2 shows why improved credit access can move child labor in either direction. The factor A^*_ℓ again determines whether the financial channel matters for labor. If $A^*_\ell = 0$, improved credit access alters consumption and welfare but does not change the marginal value of child labor. If $A^*_\ell > 0$, improved credit access has two opposing effects.

The first effect is repayment relief due to the lowered debt burden. The amount of debt exposed to the interest-rate reduction is $b^* = c^* - x$. When b^* is large, lowering r_b substantially improves terminal resources holding current choices fixed. This lowers the marginal value of harvest cashflow and pushes child labor down.

The second effect is debt-financed consumption expansion. The term $T_u(c^*) = \frac{u'(c^*)}{-u''(c^*)}$ is the household's local absolute risk tolerance for current consumption. It is measured in the same cash units as b^*. A larger $T_u(c^*)$ means that marginal utility changes slowly around c^*, so the household is willing to adjust consumption more when credit becomes cheaper. That higher consumption must be financed by additional borrowing, which raises repayment pressure and makes harvest cashflow, and therefore child labor, more valuable.

Thus the comparison $b^* \gtrless T_u(c^*)$ identifies which effect dominates. If $b^* \geq T_u(c^*)$, improved credit access primarily relieves repayment pressure and child labor falls. If $b^* < T_u(c^*)$, improved credit access primarily supports new debt-financed consumption, and child labor rises.

Regardless of the ambiguity in the child-labor response, improved credit access strictly increases consumption and welfare.

The following corollary shows that, within the borrowing regime, the effect of credit access on child labor can be organized by a single threshold on the household's initial cash position x.

Corollary 1 **(Heterogeneous Effect).** *Suppose T_u is increasing in c. Fix all parameters except x and consider the set of cash position values wherein the optimal solution is interior and the household is in the borrowing regime, $\mathcal{X}_b := \{x : c^*(x) > x\}$. Define $\psi(x) := b^*(x) - T_u(c^*(x))$. Then ψ is strictly decreasing on $\mathcal{X}_b$ and therefore changes sign at most once. In particular, there exists a threshold $\bar{x}$ on the cash position (possibly $-\infty$ or $+\infty$) such that improved credit-access (a decrease in r_b) decreases child labor if $x < \bar{x}$ and increases child labor use if $x > \bar{x}$.*

Corollary 1 shows how the household's initial cash position x acts as a critical moderator for the effect of improved credit access on the household's child labor use. As x increases within the borrowing region, the borrowed amount $b^*(x) = c^*(x) - x$ strictly falls, while $c^*(x)$ weakly rises. If T_u is increasing, then the consumption-expansion margin $T_u(c^*(x))$ weakly rises. Hence $\psi(x) := b^*(x) - T_u(c^*(x))$ is strictly decreasing in x. Low-cash borrowers tend to have a large borrowed amount and, under increasing T_u, a lower consumption-expansion margin, so credit improvements mainly act as repayment relief. Higher-cash borrowers have smaller outstanding borrowing and greater capacity to expand consumption, so credit improvements can instead induce debt-financed spending and increase child labor.

The result uses the mild monotonicity condition that $T_u(c)$ is weakly increasing. This corresponds to the standard requirement that the utility function $u(c)$ should exhibit decreasing absolute risk aversion (DARA). Economically, this means that households with higher current consumption are more willing to absorb larger absolute changes in consumption when borrowing terms improve. Several common utility functions satisfy this condition, including the constant relative risk-aversion utility (CRRA) or iso-elastic utility $u(c) = \frac{c^{1-\rho}}{1-\rho}$ for $\rho \neq 1$, the logarithmic utility $u(c) = \log(c)$, any hyperbolic absolute risk aversion (HARA) utility function with increasing linear risk tolerance, or the exponential utility $u(c) = a - e^{-bc}$ for $a, b > 0$.

4.3. Savings Access

Improved access to savings strengthens the household's net pre-harvest financial position without imposing a repayment obligation, and we model it as an increase in the savings return r_s. Below, we formalize how an increase in r_s affects child labor use, consumption, and welfare.

Proposition 3 **(Savings access)**. *Consider a range of values of r_s for which an interior optimal solution exists and the household is in the saving regime. Then,*

(i) If labor and consumption are locally strict complements ($A^*_\ell > 0$), an increase in r_s strictly decreases child labor, $\frac{\partial \ell^*}{\partial r_s} < 0$. Otherwise, child labor is unaffected by a change in r_s, $\frac{\partial \ell^*}{\partial r_s} = 0$.

(ii) The effect of an increase in r_s on consumption is generally ambiguous. More precisely,

$$\frac{\partial c^*}{\partial r_s} \leq 0 \quad \Longleftrightarrow \quad r_s s^* \left(\mathbb{E}[-v''(\bar{Y}^*)] - \frac{\left(\mathbb{E}[-v''(\bar{Y}^*)\Phi^*_\ell]\right)^2}{-\bar{V}^*_{\ell\ell}} \right) \leq \mathbb{E}[v'(\bar{Y}^*)].$$

(iii) Household welfare $\bar{V}^*(r_s)$ is strictly increasing in r_s.

Proposition 3 shows that savings access has a one-sided child-labor effect in the saving regime. A higher savings return raises the value of resources the household has already set aside. This strengthens the household's terminal financial position without creating new repayment obligations and without raising the marginal product of child labor. The marginal continuation value of harvest cashflow therefore falls, which decreases child labor. As in the cash-transfer result, the strength of the labor response is governed by $A^*_\ell = \mathbb{E}[-v''(\bar{Y}^*)\Phi^*_\ell]$. If $A^*_\ell > 0$, the marginal value of child labor falls strictly and child labor strictly decreases. If $A^*_\ell = 0$, the financial channel is moot and the higher savings return has no first-order effect on child labor.

The effect on consumption is ambiguous. A higher savings return makes postponing consumption more attractive, which pushes current consumption down. But this also raises the return on resources the household would already save, making the household effectively wealthier and pushing current consumption up. The condition in Proposition 3 characterizes when the substitution effect toward saving dominates the wealth effect, after accounting for any labor adjustment.

Importantly, unlike improved credit access, improved savings access does not create backfire risk by increasing child labor. The asymmetry arises because credit access may induce the household to expand current consumption through additional debt, increasing future repayment pressure and raising the value of harvest cashflow, which can make child labor more attractive. A higher return on savings does the opposite: it strengthens cash resources already set aside and it finances a (potential) increase in consumption by allocating a fraction of those new resources; it thus (weakly) lowers the marginal value of harvest cashflow and child labor use.

4.4. Production Support

We finally examine interventions that offer the household subsidized inputs, agronomic training, price premiums, or other forms of production or revenue support that increase its harvest cashflow. Recall that these are modeled as an increase in the production index θ appearing in the harvest cashflow function $\Phi(\ell,\theta,\xi)$. The following result summarizes their impact.

PROPOSITION 4 **(Production support).** *Consider a range of values for θ for which the optimal solution is interior and the household is inside a given fixed financial regime (borrowing, financially neutral, or savings). Define:*

$$R_\theta^* := \mathbb{E}\left[-v''(\bar{Y}^*)\Phi_\ell^*\Phi_\theta^*\right], \qquad P_\theta^* := \mathbb{E}\left[v'(\bar{Y}^*)\Phi_{\ell\theta}^*\right], \qquad C_\theta^* := \begin{cases} 0, & c^* = x, \\ \dfrac{\bar{V}_{\ell c}^*\bar{V}_{c\theta}^*}{-\bar{V}_{cc}^*}, & c^* \neq x. \end{cases}$$

(i) Child labor use is (strictly) decreasing with θ if and only if R_θ^* is (strictly) greater than $P_\theta^* + C_\theta^*$,

$$\frac{\partial \ell^*}{\partial \theta} \leq 0 \iff R_\theta^* \geq P_\theta^* + C_\theta^* \qquad \textit{and} \qquad \frac{\partial \ell^*}{\partial \theta} < 0 \iff R_\theta^* > P_\theta^* + C_\theta^*.$$

(ii) In the financially neutral regime ($c^* = x$), consumption is unaffected by a change in θ, $\frac{\partial c^*}{\partial \theta} = 0$. In the borrowing regime and in the saving regime, $\frac{\partial c^*}{\partial \theta}$ has the same sign as:

$$\bar{V}_{\ell c}^*(P_\theta^* - R_\theta^*) + (-\bar{V}_{\ell\ell}^*)\bar{V}_{c\theta}^*.$$

In particular, consumption is increasing in θ if $P_\theta^* \geq R_\theta^*$.

(iii) Household welfare $\bar{V}^*(\theta)$ is strictly increasing in θ.

Unlike cash transfers, credit, or savings, a production intervention directly affects the household's farming productivity, thereby creating additional channels through which its consumption and child labor decisions are influenced. To appreciate these, consider the tradeoffs governing the optimal child labor use, reflected in the first-order condition (FOC-ℓ):

$$d'(\ell^*) = \mathbb{E}\left[v'(\bar{Y}^*)\Phi_\ell^*\right].$$

A marginal increase in θ changes the marginal benefit of child labor (i.e., the right-hand-side above) through three channels. First, a higher θ changes terminal resources and therefore changes the marginal continuation value $v'(\bar{Y}^*)$. This effect is exactly given as $-R_\theta^*$, where $R_\theta^* := \mathbb{E}\left[-v''(\bar{Y}^*)\Phi_\theta^*\Phi_\ell^*\right] \geq 0$ is a positive *resource-relief force*. This induces the household to reduce its child labor use: the production improvement raises terminal resources, lowers the marginal value of additional harvest cashflow, and therefore lowers the marginal benefit of child labor.

The second channel is the effect of improved productivity (higher θ) on the marginal value of child labor Φ_ℓ^*, which gives rise to a *labor-productivity force* $P_\theta^* := \mathbb{E}\left[v'(\bar{Y}^*)\,\Phi_{\ell\theta}^*\right]$. The effect of this term critically depends on whether the production intervention is complementary to child labor use. When the intervention is complementary so that $\Phi_{\ell\theta}^* \geq 0$ almost surely, the term is positive and induces the household to increase its child labor use; but when the intervention is substitutable, $\Phi_{\ell\theta}^* \leq 0$, the term is negative and induces the household to reduce child labor.

The third channel arises only for households that are either in the borrowing or in the saving regime, whose consumption differs from their initial cash position, $c^* \neq x$. For such households, a higher θ can increase current consumption, and because consumption and child labor are complementary for the household ($\bar{V}_{\ell c} \geq 0$), this can create a positive feedback that could induce the household to increase its child labor use. To that end, we define the *consumption-feedback force* as:

$$C_\theta^* := \begin{cases} 0, & c^* = x, \\ \bar{V}_{\ell c}^* \, \dfrac{\bar{V}_{c\theta}^*}{-\bar{V}_{cc}^*} & c^* \neq x. \end{cases}$$

The expression exactly captures the indirect impact on child labor due to a consumption change: for $c^* \neq x$, it is the multiplicative effect of the labor-consumption complementarity ($\bar{V}_{\ell c}^* \geq 0$) and the direct effect of production support on consumption (which, for fixed ℓ, is exactly $\frac{\bar{V}_{c\theta}^*}{-\bar{V}_{cc}^*} \geq 0$).

As part (i) states, the overall effect of production support on child labor depends on the relative magnitudes of these three forces. If the resource-relief force dominates the labor-productivity and consumption-feedback forces ($R_\theta^* > P_\theta^* + C_\theta^*$), then an increase in θ would decrease child labor. If $R_\theta^* = P_\theta^* + C_\theta^*$, the intervention has no effect on child labor; if $R_\theta^* < P_\theta^* + C_\theta^*$, child labor increases. For households in the financially neutral regime, the consumption force vanishes ($C_\theta^* = 0$), so the comparison is between the resource-relief force and the labor-productivity force. The overall effect depends on – but is not solely determined by – whether the intervention is complementary to or a substitute for child labor. (For households in the financially neutral regime, any intervention that is not strictly complementary to child labor would reduce child labor, i.e., $\bar{V}_{\ell\theta}^* \leq 0$ implies $\frac{\partial \ell}{\partial \theta} \leq 0$.)

The effect of production support on consumption is more nuanced: consumption is unchanged in the financially neutral regime, but may either increase or decrease in the borrowing and saving regimes. In this regime, production support directly raises terminal resources and therefore supports higher current consumption, but this effect can be offset if the household reduces child labor and thereby gives up harvest cashflow. Consumption increases when the direct production gain is not offset by the labor adjustment; a sufficient condition is that the labor-productivity force weakly dominates the resource-relief force, $P^*_\theta > R^*_\theta$.

Part (iii) confirms that production support – which strictly increases the harvest cashflow – also strictly increases the household's welfare.

Corollary 2 **(Heterogeneous effect)**. *Suppose harvest cashflow is $\Phi(\ell,\theta,\xi) = \xi F(\ell,\theta)$, where F is twice differentiable and $F > 0$, $F_\ell > 0$, $F_\theta > 0$, $F_{\ell\ell} \le 0$, and consider an interval X_θ of cash positions x over which problem* (3) *admits an interior optimal solution. Then:*

(a) If $F_{\ell\theta} \le 0$, production support weakly reduces child labor for every cash position x, $\partial\ell^*/\partial\theta \le 0$, and the reduction is strict if $F_{\ell\theta} < 0$.

(b) If $F_{\ell\theta} > 0$, define the relative labor-complementarity of production support $K(\ell,\theta) := \frac{F(\ell,\theta)F_{\ell\theta}(\ell,\theta)}{F_\ell(\ell,\theta)F_\theta(\ell,\theta)}$ and the relief-capacity threshold $\mathcal{B}_{\hat\rho}(F,q,U)$ as:

$$\mathcal{B}_{\hat\rho}(F,q,U) := \frac{F\left\{\mathbb{E}[-v''(F\xi+\hat\rho q)\xi^2] - \dfrac{\hat\rho^2\left(\mathbb{E}[-v''(F\xi+\hat\rho q)\xi]\right)^2}{U+\hat\rho^2\mathbb{E}[-v''(F\xi+\hat\rho q)]}\right\}}{\mathbb{E}[v'(F\xi+\hat\rho q)\xi]},$$

where $\hat\rho(c;x) = r_b\mathbf{1}\{c > x\} + r_s\mathbf{1}\{c < x\}$ is an effective rate[3], $q := x - c$, and $U := -u''(c)$. Then:

(i) The child-labor response to a marginal increase in θ is given by:

$$\frac{\partial\ell^*(\theta,x)}{\partial\theta} \le 0 \quad\Longleftrightarrow\quad \Delta_\theta(x) := K\left(\ell^*,\theta\right) - \mathcal{B}^*(x) \le 0 \;\forall x \in X_\theta,$$

where $\mathcal{B}^*(x) := \mathcal{B}_{\hat\rho^*}\left(F^*,q^*,U^*\right)$, and the response is strict whenever the inequality is strict.

(ii) Recalling thresholds x_1, x_2 from Proposition 1, the child labor effect $\frac{\partial\ell^*}{\partial\theta}$ only changes side in the (closed) borrowing or saving regimes, i.e., for $x \in (-\infty, x_1] \cup [x_2, +\infty)$.

(iii) Assume the following regularity conditions hold: K is decreasing in ℓ, $\mathcal{B}_{\hat\rho}$ is increasing in F and strictly decreasing in q for $\hat\rho \in \{r_s, r_b\}$, and $u''(c)$ is increasing in c. Then:

- *the gap $\Delta_\theta(x)$ is strictly increasing in x on $(-\infty, x_1) \cup (x_2, \infty)$, is constant in x on (x_1, x_2), and exhibits a downward jump at x_1 and an upward jump at x_2*

[3] $\hat\rho(c;x)$ should not be confused with $\rho(c;x) \in \mathcal{R}(c;x)$ appearing in FOC-c; the two rates are identical in the borrowing and saving regimes, but $\hat\rho = 0$ whereas $\rho^* \in [r_s, r_b]$ in the financially neutral regime.

- *there exist thresholds x'_1, x'_2, x'_3 with $x'_1 \in [-\infty, x_1]$, $x'_2 \in \{x_1, x_2\}$, $x'_3 \in [x_2, +\infty]$, such that:*

$$\frac{\partial \ell^*}{\partial \theta} < 0 \text{ if } x \in (-\infty, x'_1) \cup (x'_2, x'_3) \qquad \text{and} \qquad \frac{\partial \ell^*}{\partial \theta} > 0 \text{ if } x \in (x'_1, x'_2) \cup (x'_3, +\infty).$$

- *if all households in the borrowing regime reduce child labor use, then so do all households in the financially neutral regime: $x'_1 = x_1 \Rightarrow x'_2 = x_1$;*
- *if households in the financially neutral regime increase child labor, then so do all households in the saving regime: $x'_2 = x_2 \Rightarrow x'_3 = x_2$.*

Corollary 2 specializes Proposition 4 to a case where uncertainty ξ affects harvest cashflow multiplicatively – a natural restriction if the uncertainty is in yield, gross margin, or their product.

Part (i) of the corollary covers production support that is a (weak) substitute for labor, $F_{\ell\theta} \le 0$. In that case, production support does not raise the marginal value of child labor. The resource-relief and labor-productivity channels therefore both push toward lowering child labor, and the multiplicative structure ensures that the consumption-feedback channel cannot overturn this force. Thus production support reduces child labor for every cash position x.

The more delicate case is when production support is complementary to labor, $F_{\ell\theta} > 0$, in which case the labor-productivity force is conducive to an increase in child labor. Under the multiplicative harvest cashflow model, the three-force comparison in Proposition 4 can now be summarized by two objects: a deterministic measure of the relative labor-complementarity of production support $K(\ell, \theta)$, and the household's relief-capacity threshold $\mathcal{B}^*(x)$. Production support will reduce child labor exactly when $K(\ell^*, \theta) \le \mathcal{B}^*(x)$.

The index $K(\ell, \theta)$ measures this complementarity in relative terms: it is the increase in the marginal value of child labor normalized by both the direct effect of the intervention and the marginal value of labor.[4] A larger K means that the intervention behaves more like a labor-productivity expansion and less like pure resource-relief.

The threshold $B^*(x)$ measures how much relative labor-complementarity the household can absorb before production support raises child labor. A larger $B^*(x)$ means that the household has greater relief capacity: the production gain can be relatively labor-complementary and still reduce child labor because harvest-resource relief remains strong. A smaller $B^*(x)$ means that the same intervention is more likely to operate as an expansion opportunity. In the borrowing and saving regimes, $\mathcal{B}^*(x) = \mathcal{B}_{\hat{\rho}}(F^*, q^*, -u''(c^*))$ also depends on the rate $\hat{\rho} \in \{r_b, r_s\}$, on the household's

[4] An equivalent interpretation views K as a ratio of two elasticities: $K(\ell, \theta) = \frac{\partial \log F_\ell}{\partial \log \theta} / \frac{\partial \log F}{\log \theta}$: how does the intervention increase the *marginal* output of child labor relative to how much it increases the total output of child labor.

pre-harvest cash position $q = x - c$, and on the curvature of current-consumption utility. In contrast, $\mathcal{B}^*(x)$ is constant in the financially neutral regime because $c^*(x) = x$ and the optimal labor decision is independent of x.

The monotonicity assumptions in part (b)-(iii) of the corollary are not needed for the pointwise comparison $K(\ell^*, \theta) \leq B^*(x)$, but rather for making global statements about how the cash position x moderates the effect of the production support intervention. The condition $K_\ell \leq 0$ requires that production support becomes relatively more labor-complementary at lower levels of child labor. This holds for many standard production functions considered in economics, including separable/Cobb-Douglas ($F(\ell,\theta) = f(\ell)g(\theta)$), constant elasticity of substitution ($F(\ell,\theta) = (a\ell^\sigma + b\theta^\sigma)^{1/\sigma}$ for $a, b > 0$ and $\sigma < 1$), or trans-log ($\log F(\ell,\theta) = \alpha \log \ell + \beta \log \theta + \gamma \log \ell \log \theta$.) Since Proposition 1 implies that child labor ℓ^* weakly decreases as x increases in the non-neutral regimes, $K_\ell \leq 0$ makes the labor-productivity force stronger for households with larger cash position x.

The conditions on $\mathcal{B}_{\hat{p}}$ impose economically intuitive requirements on the intervention's resource-relief force. A larger production base F creates more harvest-resource relief, so $\mathcal{B}_{\hat{p}}$ is assumed increasing in F; and a stronger pre-harvest cash position $q = x - c$ makes harvest-resource relief less valuable, so $\mathcal{B}_{\hat{p}}$ is assumed decreasing in q. Lastly, requiring $u''(c)$ increasing in c is a weak condition (this readily holds if the absolute consumption risk tolerance $T_u(c)$ is increasing in c, as required in Corollary 1.) Combining these assumptions with Proposition 1 ensures that a larger cash position x decreases the harvest-resource relief threshold $\mathcal{B}^*(x)$.

The financially neutral regime creates the only additional complication. Within that region the production-support response is constant in x: financially neutral households either all reduce child labor, all increase it, or all have zero response. At the lower boundary x_1, the consumption-feedback term disappears, so the sign index $\Delta_\theta(x)$ can jump downward. At the upper boundary x_2, the feedback term reappears, so the sign index can jump upward. Thus, under the stated monotonicity conditions, the child-labor response to a production support intervention can follow the sign pattern $-$, $+$, $-$, $+$ as cash position x rises, with some intervals empty. Production support may reduce child labor for the most constrained borrowers, increase it for borrowers closer to the neutral region, reduce it again for financially neutral or low-saving households, and increase it for sufficiently high-liquidity savers. The corollary also identifies simpler cases. If production support does not backfire among any borrowers, then it also does not backfire in the financially neutral regime (and it can only backfire for households in the saving regime). If it increases child labor in the financially neutral regime, then it also increases child labor throughout the saving regime and the only child-labor reducing effects can be observed among households that borrow.

Overall, Corollary 2 sharpens the interpretation of production support: when this weakly substitutes labor, it reduces child labor for all types of households; but when it complements child labor, the effect can vary with the household's cash position: for low-cash households it is more likely to act as harvest-resource relief and reduce child labor, whereas for higher-cash or higher-expansion-capacity households it is more likely to act as a production or consumption expansion opportunity. This is the production-side analogue of the credit result: the same nominal intervention can act as relief for one household and as expansion for another.

5. Setting, Data, and Empirical Strategy

The comparative statics in §4 have shown that intervention effects can be heterogeneous and are moderated by the household's cash position x, which determines the household's financial regime.

We next examine whether and how these heterogeneous effects appear in household-level survey data collected by an NGO partner from cocoa-growing households in Ghana. Our NGO partner is a nonprofit foundation that works to prevent and remediate child labor and forced labor in cocoa-growing communities. As part of this work, it conducts large-scale surveys of cocoa-producing households in Ghana and Côte d'Ivoire. The surveys record information on household demographics, finances, farming operations, child-labor outcomes, and the poverty-reduction or child-labor interventions received by each household.

The survey data are valuable because they cover a population and a set of characteristics that are difficult to observe systematically. Cocoa-producing households often operate in remote areas and through informal first-mile value chains, so accurate administrative records of their production and resource use are limited. Moreover, governmental living-standard surveys tend to underestimate the prevalence of child labor due to social-desirability biases; in contrast, our NGO partner's surveys employ carefully designed protocols to assess child labor by conducting interviews with both parents and children, which improves response consistency and reduces such biases.

Nevertheless, the nature of the survey data also limits what our empirical analysis can claim. Many household variables are self-reported and cannot be externally verified. Critically, the intervention variables are imperfect proxies of the interventions discussed in our model, capturing endogenous participation in cash-transfer programs or access to affordable loans rather than randomized interventions, and some relevant household characteristics are measured contemporaneously rather than at strict pre-intervention baselines. We therefore set a modest empirical goal for the subsequent analysis: we estimate adjusted associations between these intervention proxies and child labor outcomes rather than proper causal treatment effects, and we use these estimates primarily to assess whether the observable patterns are consistent with the model's theoretical predictions.

5.1. Treatment, Outcome, and Operational State Variables

Our analysis uses two household-level datasets from two distinct surveys, focused on cocoa-growing households in Ghana. The surveys do not experimentally vary the model primitives x, r_b, r_s, or θ, so we adopt empirical variables that act as close proxies for the variables of interest in our model. Because the intervention proxies, the child-labor outcomes, and the household features differ in the two datasets, we analyze the datasets separately. The first dataset ("Dataset 1") includes 640 households and supports analyses related to cash transfers and credit access. The second dataset ("Dataset 2") includes 834 households and supports analysis of production support. The data lack good proxy measures for improved access to savings, so we do not consider these interventions. We subsequently discuss all the proxy variables used, and Table 1 contains a concise summary.

In Dataset 1, the "cash-transfer status" variable records whether the household participated in a cash transfer program during the previous year, and is the closest proxy to an increase in the household's cash position x. The variable should still be interpreted cautiously: it records whether the household is listed as a cash recipient, but not the exact timing, size, or use of the transfer.

Dataset 1 also records a household's "loan-repayment ability," which indicates whether the household is able to make its next loan repayment. We use this as an imperfect empirical proxy to improved credit access (r_b-proxy) because households able to make repayments are likely to face lower effective repayment pressure or better borrowing terms. This is reported only by households that also report having an outstanding loan, so the variable is missing for 372 of the 640 households. Although this proxies for repayment capacity, the variable is obviously not equivalent to the model's improvement in credit access by reducing the borrowing rate r_b. A household's ability to repay its loan critically depends on the loan size, which is unfortunately not recorded in the data. Moreover, our analysis only relies on the subsample corresponding to households that report having an outstanding loan, which may be a biased subsample. (Households without outstanding loans may either not need loans or may not have been able to secure loans, so removing those records may create inherent bias.)

In Dataset 1, several variables can serve as proxies for outcomes of interest related to child labor (ℓ-proxies). Specifically, the data include two binary indicators recording whether at least one child in the household was engaged in labor (i) over the past seven days, and (ii) over the past six months.

Table 1 Main empirical variables

Variable	Role	Dataset	Interpretation	Nonmissing	Mean
Cash-transfer status	Treatment proxy	Dataset 1	Indicator for whether the household is recorded as a cash recipient, proxy for a cash transfer shift.	640	0.422
loan-repayment ability	Treatment proxy	Dataset 1	Indicator for whether the household reports being able to make the next loan repayment, proxy for repayment capacity, observed only for households with outstanding loans.	268	0.679
Child labor, past 7 days	Outcome	Dataset 1	Indicator for whether at least one child in the household engaged in child labor during the past seven days.	640	0.581
Child labor, past 6 months	Outcome	Dataset 1	Indicator for whether at least one child in the household engaged in child labor during the past six months.	640	0.750
Income per capita per day	Cash-position proxy	Dataset 1	Proxy for household resource position.	638	5.734
Spending per capita per day	Cash-position proxy	Dataset 1	Proxy for realized consumption pressure and short-run household needs.	640	6.389
Potential production-improving benefits	Treatment proxy	Dataset 2	Indicator for whether the household's potential program-benefit list includes a production-improving category.	834	0.591
Obtained production-improving benefits	Treatment proxy	Dataset 2	Indicator for whether the household reports obtaining a production-improving benefit.	834	0.590
Any household child labor	Outcome	Dataset 2	Indicator for whether the household reports any positive child days in cocoa work.	834	0.729
Revenue per capita per day	Cash-position proxy	Dataset 2	Proxy for cocoa production scale.	834	2.852
Net income per capita per day	Cash-position proxy	Dataset 2	Proxy for realized surplus after production costs.	834	1.244

Because Dataset 1 does not directly record a measure of the household's cash position x, we use two variables to proxy for this key moderator: (i) the household's income per capita per day, and (ii) the household's spending per capita per day. Both are derived by suitably normalizing corresponding variables in the data, which record total yearly household income and household expenditure during the last month, respectively.[5] Both variables are imperfect measures of the household's (liquid) cash position, but are likely correlated with that measure; and the availability of directly reported

[5] The reported expenditure covers food, healthcare, housing/rent, education, clothing/shoes, religious and social commitments, and other expenses. We normalize by the total number of individuals in the household, but normalizing by the male adult-equivalent value (which we can compute because we have access to the household composition) does not significantly change the results.

consumption is rare and particularly valuable as a measure of realized consumption, which is positively related to x in our model.

Dataset 2 contains two variables that are reasonable proxies for interventions offering production support. Specifically, those data records (i) whether the household participated in a program that listed a production-improvement as a potential benefit, and (ii) whether the household reports having obtained production benefits from such program participation. These variables are useful because they are directly tied to the farm's production, but they should not be interpreted as randomized productivity shocks: they are observed program-benefit proxies, and they may combine several types of support that affect production through different mechanisms.

The child labor outcome of interest in Dataset 2 is whether the household reports any positive number of child days in cocoa work over the previous crop year.

Lastly, the proxy variables for cash position x in Dataset 2 are (i) revenue per capita per day and (ii) net income per capita per day, also derived by suitably normalizing annualized quantities.[6] Dataset 2 does not contain variables related to discretionary expenditure, so we cannot include a proxy for x based on these.

5.2. Empirical Strategy

Our goal is to quantify the impact of each binary intervention proxy (for $\eta \in \{x, r_b, \theta\}$) on the child labor proxy variable (ℓ-proxy), as a function of the household's cash position (x-proxy). To facilitate the exposition by avoiding obscure terminology and relating to known methods, we subsequently describe this as the task of estimating heterogeneous treatment effects, but we again caution *against* interpreting our results as causal effects or statements. We thus use the term "treatment effect" below to describe the estimated difference in child-labor outcomes between households with and without a given intervention proxy, conditional on relevant covariates of interest.

For each triple of (treatment, outcome, cash-position proxy) defined in §5.1 and summarized in Table 1, we estimate heterogeneous treatment effects using a causal forest estimator (Wager and Athey 2018, Athey et al. 2019). This allows the treatment effects to vary flexibly with household characteristics. We first form a complete sample for the relevant triple and any controls in the relevant survey module. Details on the control variables can be found in Appendix §EC.1. Categorical data are expanded into indicator variables. We fit a generalized random forest with the binary intervention proxy as the treatment, the binary child-labor measure as the outcome, and the cash-position proxy

[6] We derive annual revenue as the average revenue from the 2011/2012 and 2013/2014 cocoa seasons, omitting missing values. Net income subtracts annual costs for hired labor and input costs for fertilizer, fungicide, and insecticide from the annual revenue.

and other relevant controls as covariates. For each fitted forest, we compute subgroup average treatment effects using the forest's doubly robust augmented inverse-propensity-weighted (AIPW) scores, with subgroups defined by the relevant cash-position proxy.

To calculate heterogeneous effects, we average the treatment effects over "poorer" and "richer" households defined in terms of the lower- and upper-tails of the marginal distribution of the cash-position proxy, respectively. We consider several definitions of "poorer/left-tail" subgroups, corresponding to all households whose cash-proxy variable is below the 20th, 30th, 40th, and 50th percentiles, but also below certain external poverty and living-income benchmarks. We consider the World Bank $2.15/day poverty line in 2017 PPP, a 2018 food-plus-housing benchmark of 955 GHS/month, and a 2018 living-income benchmark of 1,464 GHS/month, all normalized to the appropriate per-capita daily scale and adjusted to the relevant dataset year. Details on the conversion procedure and source values are provided in Appendix §EC.1. Similarly, we define "richer/right-tail" subgroups as containing households whose cash-position proxy variable takes a value at or above the 50th, 60th, 70th, and 80th percentiles. The figures plot all the quantile and threshold estimates together to facilitate comparisons between "poorer" and "richer" households (based on the cash-position proxy variable). Since the child labor outcomes are binary, estimates are percentage-point differences in the probability that a household used child labor. The reported bars are pointwise 95 percent Wald intervals based on the generalized random forest augmented inverse-propensity-weighted standard errors.

The main body reports results based on the proxy variables most closely tied to the model. As robustness checks, we re-estimated the same analyses after changing how the cash-position proxy was constructed or how the sample was defined. First, we used adult-male-equivalent normalization instead of household-size normalization. Second, we used the underlying unnormalized variables directly, such as total annual income, monthly expenditure, annual revenue, and annual net income. Third, for the credit-related specifications, we checked whether the results changed when restricting the sample to households with outstanding loans, households without outstanding loans, or households whose spending exceeded income. Finally, we checked whether extreme values of the cash-position proxy drove the results by either dropping or winsorizing observations above the 95th or 99th percentile. The resulting qualitative patterns are unchanged; we do not report these additional estimates for space reasons.

6. Empirical Patterns

The results below describe the heterogeneous associations around the three intervention proxies.

6.1. Pattern 1: Cash-Recipient Status

Figure 1 reports the estimated treatment effects for the cash transfer proxy (cash-recipient status) on the child labor proxy, for subgroups defined by either of the two cash-position proxy variables (income or spending per capita per day). The results are consistent with the theory developed in Proposition 1: estimated effects are close to zero (and insignificant) or negative for both poorer/left-tail and richer/right-tail subgroups. Moreover, some of the plots also exhibit the heterogeneity pattern predicted by Proposition 1, whereby the cash transfer effect is near zero/insignificant for subgroups with lower values of the cash-position proxy, but it is negative and more significant for the subgroups with higher values.

6.2. Pattern 2: Repayment Capacity

Similarly, Figure 2 reports corresponding estimates for the effect of the improved-credit-access proxy (loan-repayment ability) on the child labor proxy, for subgroups based on the same two cash-position proxy variables (income or spending per capita per day). The results are consistent with the theory developed in Proposition 2 and Corollary 1: estimated effects are dependent on the cash-position proxy and are close to zero or negative (and possibly significant) for households with lower values of the cash-position proxy, whereas they are positive (and possibly significant) for households with larger values of the cash-position proxy.

6.3. Pattern 3: Production Support Show the Clearest Expansion Pattern

Lastly, Figure 3 reports the estimates for the effect of a production support proxy (potential or obtained program benefits) on the child labor proxy, for subgroups based on the two cash position proxies derived from Dataset 2 (net income or revenue, per capita, per day). The results are consistent with the theoretical predictions in Proposition 4 and Corollary 2: estimated effects depend on the cash-position proxy and are close to zero or slightly negative (but not significant) for households with lower cash positions, whereas they are positive (and possibly significant) for households with higher cash positions.

6.4. Synthesis

Taken together, the three patterns align with the model's central results. The cash-transfer (proxied by cash-transfer status) shows larger, potentially significant reductions in child labor among households with higher cash-position proxy, but no significant effect in households with lower cash-position proxy. The improved credit access (proxied by a self-reported ability to repay loans) shows larger and potentially significant reductions in child labor for households with lower cash-position proxy,

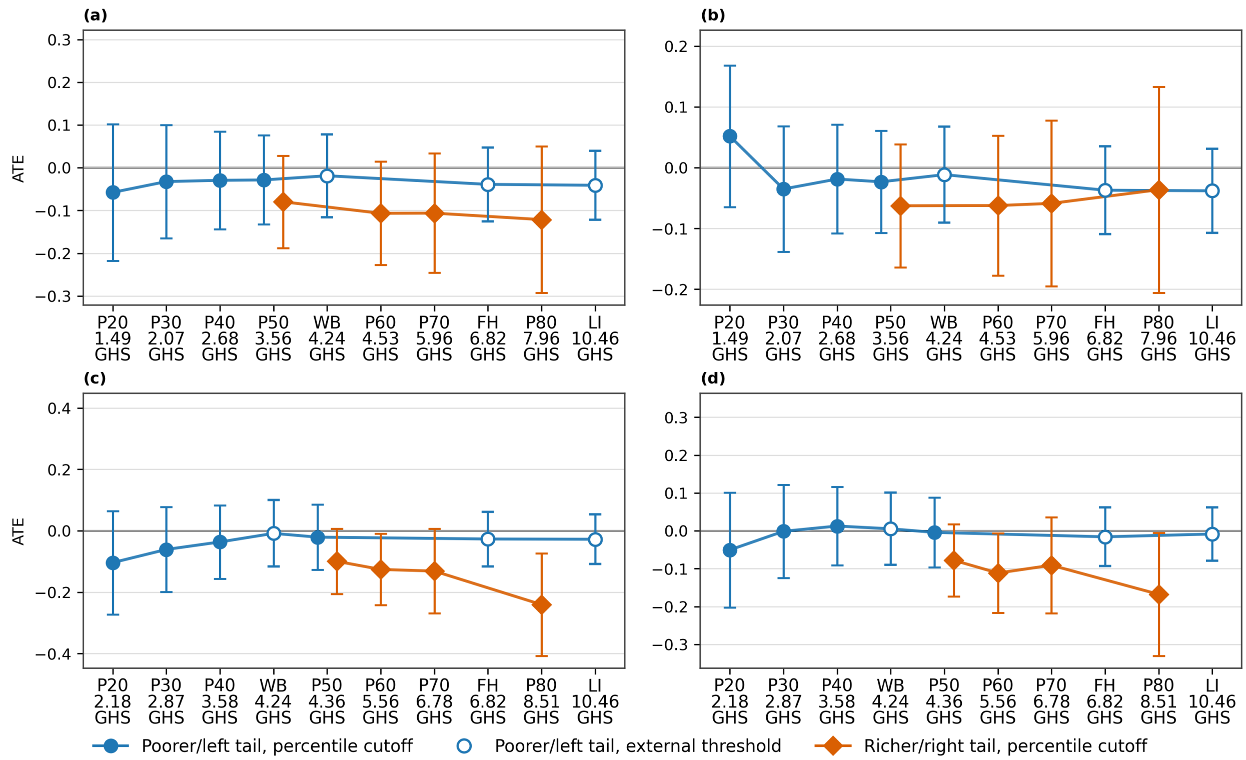

Figure 1 **Heterogeneous effects of cash transfers on household child labor. The outcome variable (ℓ-proxy) is whether child labor was used during the past 7 days (panels a,c) or during the past 6 months (panels b,d). The cash-position proxy (x-proxy) is the income per capita per day (panels a,b) and consumption per capita per day (panels c,d). Tick labels show percentile cutoffs (P) or monetary external thresholds (WB = World Bank 2.15 USD/day, FH = food-plus-housing, LI = living-income, all converted in Ghanaian Cedis per person per day). Blue circles denote "poorer/left-tail" subgroups of households whose x-proxy variable is below a cutoff defined using percentiles (for full circles, with P indicating the percentile) or using external cutoffs (for hollow circles). Filled orange diamonds denote "richer/right-tail" subgroups of households whose x-proxy variable is above the indicated percentile cutoff. Bars are 95 percent confidence intervals, estimated using a causal forest estimator and augmented inverse-propensity-weighted (AIPW) scores.**

but leads to no effect (or even a positive effect) in households with higher cash positions. Lastly, production support (proxied by participation in a program with potential or obtained production benefits) shows potential to reduce child labor for households with lower cash positions, but not for those with larger cash positions.

7. Conclusion

Responsible-sourcing interventions affect child labor through three forces: resource relief, which lowers the marginal value of harvest cashflow and reduces child labor; a labor-productivity effect, which raises child labor when the intervention complements farm production; and a consumption-expansion effect, which raises child labor when the intervention finances current consumption

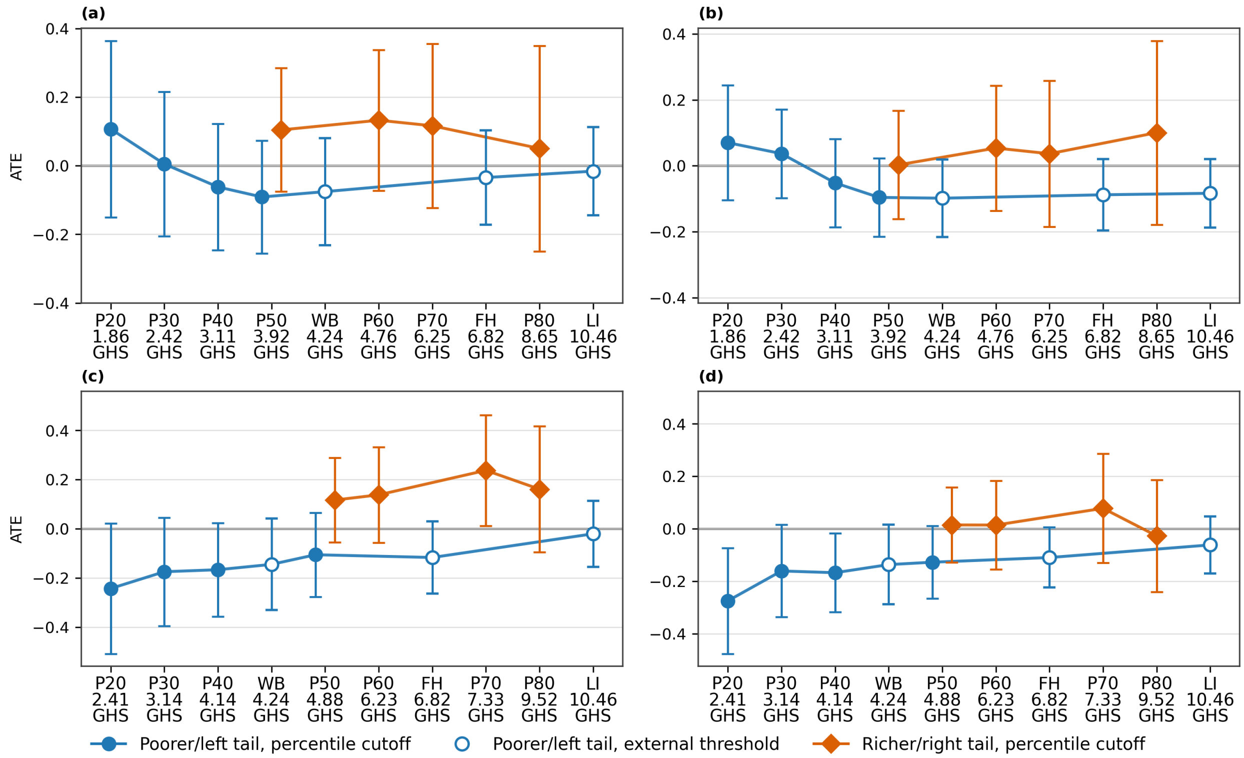

Figure 2 **Heterogeneous effects of improved credit access proxy on household child labor proxy. The outcome variable (ℓ-proxy) is whether child labor was used during the past 7 days (panels a,c) or during the past 6 months (panels b,d). The cash-position proxy (x-proxy) is the income per capita per day (panels a,b) and consumption per capita per day (panels c,d). Tick labels show percentile cutoffs (P) or monetary external thresholds (WB = World Bank 2.15 USD/day, FH = food-plus-housing, LI = living-income, all converted in Ghanaian Cedis per person per day). Blue circles denote "poorer/left-tail" subgroups of households whose x-proxy variable is below a cutoff defined using percentiles (for full circles, with P indicating the percentile) or using external cutoffs (for hollow circles). Filled orange diamonds denote "richer/right-tail" subgroups of households whose x-proxy variable is above the indicated percentile cutoff. Bars are 95 percent confidence intervals, estimated using a causal forest estimator and augmented inverse-propensity-weighted (AIPW) scores.**

that must be repaid from harvest cashflow. Cash transfers and savings support operate primarily through resource relief and carry low backfire risk. In contrast, credit access, which also activates consumption expansion, and production support, which activates all three forces, can increase child labor among households. Household survey data from Ghana are consistent with these patterns across all three intervention classes. The gradient is sharpest for production support interventions, which are associated with lower child labor among constrained households and higher child labor among better-positioned ones.

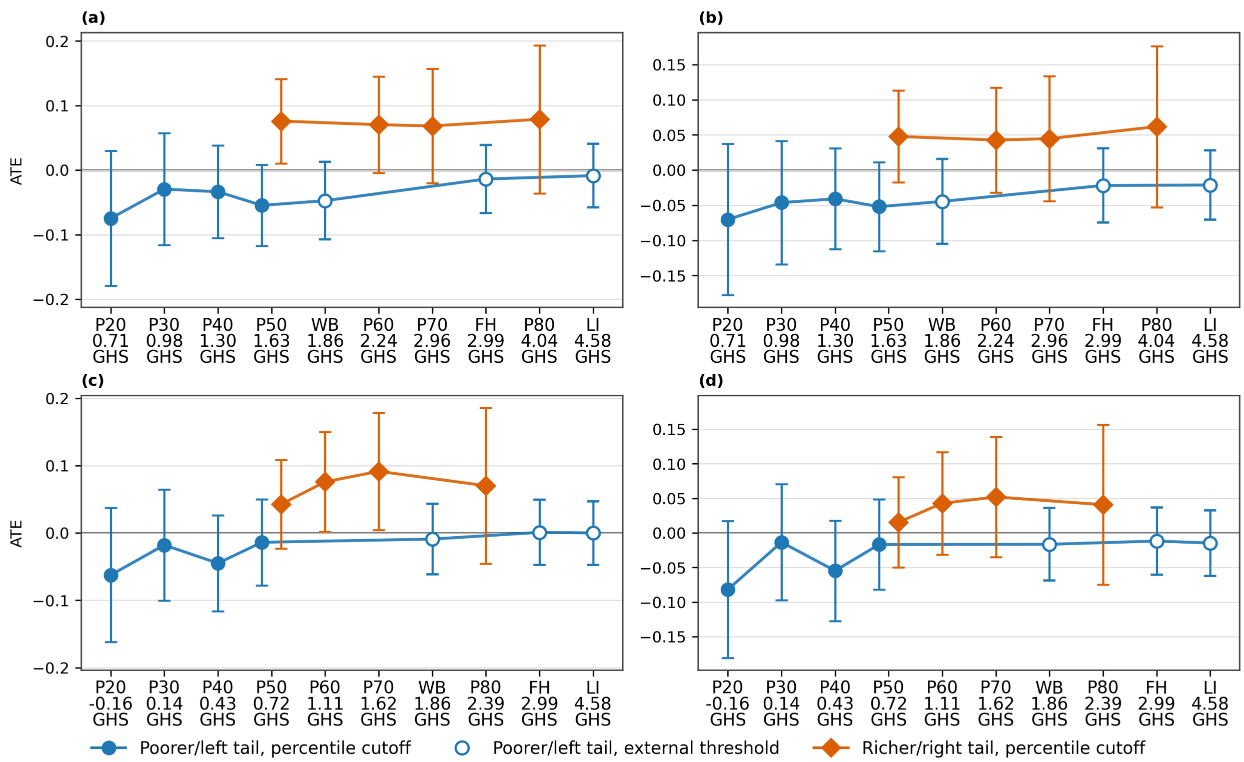

Figure 3 **Heterogeneous effects of production support on household child labor proxy. The outcome variable (ℓ-proxy) is whether any child labor was used during the past harvest cycle. The cash-position proxy variable (x-proxy) is the revenue per capita per day (a,b) or the net income per capita per day (c,d). Treatments are whether the household participated in a program with potential production benefits (a,c) or whether the household obtained production benefits from such participation (b,d). Tick labels show percentile cutoffs (P) or monetary external cutoffs (WB = World Bank 2.15 USD/day, FH = food-plus-housing, LI = living-income, all converted in Ghanaian Cedis per person per day). Blue circles denote "poorer/left-tail" subgroups of households whose x-proxy variable is below a cutoff defined using percentiles (for full circles, with P indicating the percentile) or using external cutoffs (for hollow circles). Filled orange diamonds denote "richer/right-tail" subgroups of households whose x-proxy variable is above the indicated percentile cutoff. Bars are 95 percent confidence intervals, estimated using a causal forest estimator and augmented inverse-propensity-weighted (AIPW) scores.**

For practitioners implementing responsible-sourcing programs at scale, the implication is that targeting interventions based on household characteristics already measured in standard monitoring surveys – income, spending pressure, repayment capacity, and production scale – can meaningfully reduce backfire risk. At the same time, investments in richer longitudinal data collection across agricultural communities would expand the frontier of what responsible-sourcing programs can achieve: better measurement of household financial positions, production environments, and welfare outcomes over time would allow practitioners and researchers alike to identify which households benefit most from which interventions, and to refine program design in ways that current cross-sectional monitoring surveys cannot support. The operations management community is well-positioned to contribute to this agenda — agricultural settings in developing economies are consequential, underexplored, and present precisely the kind of complex operational tradeoffs under uncertainty where our discipline has the most to offer.

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E-Companion

EC.1. Estimation Details

The main-text causal-forest estimates use the generalized random forest framework of Athey et al. (2019) and Wager and Athey (2018). Each forest includes the cash-position proxy used for heterogeneity and a set of controls available in the relevant survey. For Dataset 1, used in the cash-transfer and loan-repayment analyses, controls cover child characteristics, household composition, household-head demographics and education, migration status, respondent characteristics, district, disability status, and years lived in the community. For Dataset 2, used in the production-benefit analysis, controls cover respondent characteristics, household composition, cocoa cultivation, years of cocoa-farming experience, district, and travel times to local services and cocoa-market locations.

Monetary thresholds are converted separately because Dataset 1 is dated to the 2019 survey period, while Dataset 2 uses cocoa-income variables from the 2011/12 and 2013/14 crop seasons. World Bank poverty thresholds are converted using Ghana private-consumption PPP factors and CPI, $T_y = L_b \times PPP_b \times CPI_y / CPI_b$. The 2018 Ghana cocoa benchmarks, 955 GHS/month for food plus housing and 1,464 GHS/month for living income, are normalized by 30 days and the five-person reference household. No exchange-rate conversion is used. The resulting cutoffs, in the order World Bank \$2.15/day, food plus housing, and living income, are 4.2448, 6.8215, and 10.4572 GHS/person/day, for Dataset 1, and 1.8577, 2.9853, and 4.5765 GHS/person/day, for Dataset 2.

EC.2. Proofs for Section 3

Proof of Lemma 1. Fix (ℓ, c) and consider feasible choices $b \geq 0, s \geq 0$ satisfying $c + s = x + b$. If $c \leq x$, any feasible pair can be written as $s = x - c + b$ with $b \geq 0$, and the household's objective can be rewritten as: $u(c) - d(\ell) + \mathbb{E}\big[v\big(\Phi(\ell, \theta, \xi) + r_s(x - c) - (r_b - r_s)b\big)\big]$.

Because $v' > 0$ and $r_b > r_s$, the objective is maximized by setting $b^* = 0$, or equivalently, $s^* = x - c$.

If $c > x$, any feasible pair can be written as $b = c - x + s$ with $s \geq 0$, and the objective becomes:

$$u(c) - d(\ell) + \mathbb{E}\big[v\big(\Phi(\ell, \theta, \xi) - r_b(c - x) - (r_b - r_s)s\big)\big],$$

which is uniquely maximized by setting $s^* = 0$ and $b^* = c - x$. It then follows that at optimality, $b^* = (c^* - x)^+$ and $s^* = (x - c^*)^+$. □

Lemma EC.1 **(Strict concavity).** *The household's objective $\bar{V}(\ell, c; x, \theta)$ is strictly concave on the interior of the effective domain* $\text{int}(\text{dom}(d)) \times \text{int}(\text{dom}(u))$. *Consequently, interior optimal solutions are unique. Moreover, $\bar{V}$ is twice differentiable at any interior solution with $c \neq x$ and its Hessian is negative definite.*

Proof of Lemma EC.1. The function $q \mapsto r_s q^+ - r_b(-q)^+$ is increasing and concave because its slope falls from r_b for $q < 0$ to r_s for $q > 0$. Therefore $(\ell, c) \mapsto \Phi(\ell, \theta, \xi) + r_s(x-c)^+ - r_b(c-x)^+$ is concave: the production term Φ is concave in ℓ by assumption, and the financial term is concave in $x - c$. Since v is increasing and concave, the composition with v is concave for each realization of ξ, and the expectation preserves concavity. The term $u(c) - d(\ell)$ is strictly jointly concave in (ℓ, c) because u is strictly concave in c and $-d$ is strictly concave in ℓ. Hence $\bar{V}$ is strictly concave.

Our assumptions also imply that for $c \neq x$, the function $\bar{V}$ is twice differentiable because the household is either in a regime where it borrows ($c > x$) or is in a regime where it saves ($x > c$), so the Hessian is well defined and is negative definite because $\bar{V}$ is strictly concave. □

EC.2.1. Generic Expressions in a Fixed Financial Regime

For some results, it helps to fix a financial regime with rate $\rho \in \{r_b, r_s\}$. In this case, we have:

$$\bar{Y}(\ell, c, \theta, \xi) := \Phi(\ell, \theta, \xi) + \rho(x - c), \qquad \bar{V}(\ell, c) := u(c) - d(\ell) + \mathbb{E}[v(\bar{Y}(\ell, c, \theta, \xi))].$$

and both $\bar{Y}$ and $\bar{V}$ are twice differentiable in ℓ, c. The interior optimal solution (ℓ^*, c^*) is given by the first-order conditions:

$$\bar{V}_\ell(\ell^*, c^*; x, \theta) = -d'(\ell^*) + \mathbb{E}[v'(\bar{Y}^*)\Phi^*_\ell] = 0 \tag{EC.1a}$$

$$\bar{V}_c(\ell^*, c^*; x, \theta) = u'(c^*) - \rho\mathbb{E}[v'(\bar{Y}^*)] = 0. \tag{EC.1b}$$

Differentiating these with respect to any parameter $\eta \in \{x, \rho, \theta\}$ then gives:

$$\begin{pmatrix} \bar{V}^*_{\ell\ell} & \bar{V}^*_{\ell c} \\ \bar{V}^*_{\ell c} & \bar{V}^*_{cc} \end{pmatrix} \begin{pmatrix} d\ell^*/d\eta \\ dc^*/d\eta \end{pmatrix} = -\begin{pmatrix} \bar{V}^*_{\ell\eta} \\ \bar{V}^*_{c\eta} \end{pmatrix}. \tag{EC.2}$$

The relevant derivatives for all parameters of interest can be expressed as follows:

$$\begin{aligned}
&\bar{V}^*_{\ell\ell} = -d''(\ell^*) + \mathbb{E}[v''(\bar{Y}^*)(\Phi^*_\ell)^2 + v'(\bar{Y}^*)\Phi^*_{\ell\ell}] < 0, && \bar{V}^*_{cc} = u''(c^*) + \rho^2\mathbb{E}[v''(\bar{Y}^*)] < 0 \\
&\bar{V}^*_{\ell c} = -\rho\mathbb{E}[v''(\bar{Y}^*)\Phi^*_\ell] \geq 0 && \bar{V}^*_{\ell x} = \rho\mathbb{E}[v''(\bar{Y}^*)\Phi^*_\ell] = -\bar{V}^*_{\ell c} \leq 0 \\
&\bar{V}^*_{cx} = -\rho^2\mathbb{E}[v''(\bar{Y}^*)] \geq 0 && \bar{V}^*_{\ell\rho} = (x - c)\mathbb{E}[v''(\bar{Y}^*)\Phi^*_\ell] \\
&\bar{V}^*_{c\rho} = -\mathbb{E}[v'(\bar{Y}^*)] - \rho(x - c)\mathbb{E}[v''(\bar{Y}^*)] && \bar{V}^*_{\ell\theta} = \mathbb{E}[v''(\bar{Y}^*)\Phi^*_\ell\Phi^*_\theta + v'(\bar{Y}^*)\Phi^*_{\ell\theta}] \\
&\bar{V}^*_{c\theta} = -\rho\mathbb{E}[v''(\bar{Y})\Phi_\theta] \geq 0.
\end{aligned} \tag{EC.3}$$

The strict concavity of $\bar{V}$ implies that the determinant of the Hessian appearing in (EC.2) satisfies:

$$D^* := \bar{V}^*_{\ell\ell}\bar{V}^*_{cc} - (\bar{V}^*_{\ell c})^2 > 0.$$

Lastly, for any scalar parameter η, the implicit-function theorem gives

$$D^*\frac{\partial \ell^*}{\partial \eta} = V_{\ell c}V_{c\eta} - V_{cc}V_{\ell\eta}, \qquad D^*\frac{\partial c^*}{\partial \eta} = \bar{V}^*_{\ell c}\bar{V}^*_{\ell\eta} - \bar{V}^*_{\ell\ell}\bar{V}^*_{c\eta}. \tag{EC.4}$$

EC.2.2. Proofs for Results in §4.1 on Cash Transfers

Proof of Proposition 1. With $q = x - c$ denoting the household's pre-harvest financial position, the household's problem (3) can be reformulated as:

$$\bar{V}^*(x) = \max_q \left[u(x-q) + B(q)\right], \tag{EC.5}$$

where

$$B(q) := \max_\ell \left\{-d(\ell) + \mathbb{E}\left[v\big(\Phi(\ell,\theta,\xi) + f(q)\big)\right]\right\}, \qquad f(q) := r_s q^+ - r_b(-q)^+ = \begin{cases} r_b q, & q<0, \\ r_s q, & q \geq 0. \end{cases}$$

(i-iii). We prove these parts together. Because f is concave in q (as $r_b > r_s$), Φ is concave in ℓ, v is increasing and concave, and d is convex, the function that is maximized to obtain $B(q)$ is jointly concave in (q,ℓ). In turn, this implies that B is concave in q. B is strictly increasing in q because f is strictly increasing in q.

Because u and B are both concave, the function $u(x-q)+B(q)$ is supermodular in (x,q). Therefore, the optimal solution $q^*(x)$ is increasing in x. By a symmetric argument, the function $u(c)+B(x-c)$ is supermodular in (x,c), so the optimal solution $c^*(x)$ to the problem $\max_c [u(c)+B(x-c)]$ is increasing in x. We conclude that $c^*(x)$ and $q^*(x)$ are increasing in x.

At an interior optimal solution for problem (EC.5), we must have:

$$u'(x-q^*(x)) \in \partial^+ B(q^*(x)), \tag{EC.6}$$

where ∂^+ denotes the superdifferential of the concave function B.

Consider first the financially neutral region, where $c = x$ and thus $q = 0$. The optimal labor choice ℓ is found by solving the problem:

$$\max_\ell \{-d(\ell) + \mathbb{E}[v(\Phi(\ell,\theta,\xi))]\}.$$

The interior solution is the unique ℓ^* satisfying:

$$d'(\ell^*) = \mathbb{E}[v'(\Phi(\ell^*,\theta,\xi))\,\Phi_\ell(\ell^*,\theta,\xi)]. \tag{EC.7}$$

Letting $M_0 := \mathbb{E}\left[v'(\Phi(\ell^*,\theta,\xi))\right]$, note that the left and right derivatives of B at $q=0$ are

$$B'_-(0) = r_b M_0, \qquad B'_+(0) = r_s M_0.$$

Therefore, the superdifferential of B at 0 is $\partial^+ B(0) = [r_s M_0, r_b M_0]$, and by (EC.6), we have that $q^*(x) = 0$ is the optimal solution here if and only if $r_s M_0 \leq u'(x) \leq r_b M_0$.

Since u' is strictly decreasing, the set of values of x satisfying the above inequality is an interval. Let $x_1 \leq x_2$ be the endpoints of this interval. (When both are finite, these points satisfy $u'(x_1) = r_b M_0$, $u'(x_2) = r_s M_0$, but note that we can have $x_1 = -\infty$ or $x_2 = +\infty$, depending on whether the inverse of u' exists in the relevant range.) On this interval, $c^*(x) = x$ and $q^*(x) = 0$.

Moreover, the household is in a borrowing regime ($c > x, q < 0$) for $x < x_1$ and is in a saving regime ($c < x, q > 0$) for $x > x_2$, which follows from (EC.6) and because $q^*(x)$ and $x - q^*(x)$ are increasing in x.

To prove the comparative statics concerning $c^*(x), q^*(x)$ and $\ell^*(x)$ in the borrowing and saving regime, fix a point with $c^*(x) \neq x$, and let $\rho \in \{r_b, r_s\}$ denote the relevant rate. Locally,

$$\bar{Y}(\ell, c; x, \theta, \xi) = \Phi(\ell, \theta, \xi) + \rho(x - c),$$

and $\bar{V}$ is twice differentiable in (ℓ, c). In this case, the relevant comparative statics can be recovered from (EC.4) for the parameter $\eta = x$. Specifically, we obtain:

$$D^* \frac{\partial \ell^*(x)}{\partial x} = \bar{V}^*_{\ell c} \bar{V}^*_{cx} - \bar{V}^*_{cc} \bar{V}^*_{\ell x} = \bar{V}^*_{\ell c} (\bar{V}^*_{cx} + \bar{V}^*_{cc}) = u''(c^*(x)) \bar{V}^*_{\ell c}.$$

Because $u'' < 0$, this readily implies that $\frac{\partial \ell^*(x)}{\partial x} \leq 0$, with strict inequality precisely when

$$\mathbb{E}[-v''(\bar{Y}^*) \Phi_\ell(\ell^*(x), \theta, \xi)] > 0.$$

After some algebra, it can also be verified that:

$$D^* \left(1 - \frac{\partial c^*(x)}{\partial x}\right) = \bar{V}^*_{\ell\ell} (\bar{V}^*_{cc} + \bar{V}^*_{cx}) = \bar{V}^*_{\ell\ell} u''(c^*(x)) > 0,$$

because $\bar{V}^*_{\ell\ell} < 0$ and $u''(c^*(x)) < 0$. We conclude that $\frac{\partial c^*(x)}{\partial x} < 1$ and so $\frac{\partial q^*(x)}{\partial x} = 1 - \frac{\partial c^*(x)}{\partial x} > 0$. □

(iv). Because u is strictly increasing (as $u' > 0$), the function $u(x - q) + B(q)$ is strictly increasing in x, and maximization preserves the ordering, so $\bar{V}^*(x)$ must be strictly increasing in x.

EC.2.3. Proofs for Results in §4.2 on Improved Credit Access

Proof of Proposition 2. Recall from the proof of Proposition 1 that within the financial regime, $\bar{Y}(\ell, c; x, \theta, \xi) = \Phi(\ell, \theta, \xi) + r_b(x - c)$ and $\bar{V}$ is twice differentiable in (ℓ, c). For notational compactness, recall that $b^* = c^* - x > 0$ and define

$$A^*_\ell := \mathbb{E}[-v''(\bar{Y}^*) \Phi^*_\ell] \geq 0, \qquad C^* := \mathbb{E}[-v''(\bar{Y}^*)] \geq 0, \qquad M^* := \mathbb{E}[v'(\bar{Y}^*)] > 0.$$

(i). The comparative statics for ℓ^* can be recovered from (EC.4) for $\eta = r_b$. We obtain:

$$D^* \frac{\partial \ell^*}{\partial r_b} = \bar{V}^*_{\ell c} \bar{V}^*_{c r_b} - \bar{V}^*_{cc} \bar{V}^*_{\ell r_b}.$$

From (EC.3) evaluated in the borrowing regime $\rho = r_b$, we have:

$$\bar{V}^*_{\ell c} = r_b A^*_\ell, \qquad \bar{V}^*_{\ell r_b} = b^* A^*_\ell, \qquad \bar{V}^*_{c r_b} = -\big(M^* + r_b b^* C^*\big), \qquad \bar{V}^*_{cc} = u''(c^*) - r_b^2 C^*.$$

Substituting these expressions, we have:

$$D^*\frac{\partial \ell^*}{\partial r_b} = r_b A^*_\ell\left[-(M^*+r_b b^* C^*)\right] - \left(u''(c^*) - r_b^2 C^*\right) b^* A^*_\ell = -A^*_\ell\left(r_b M^* + b^* u''(c^*)\right).$$

The consumption first-order condition in the borrowing regime is $u'(c^*) = r_b\mathbb{E}[v'(\bar{Y}^*)] = r_b M^*$, which allows simplifying the expression above as:

$$D^*\frac{\partial \ell^*}{\partial r_b} = -A^*_\ell\left(u'(c^*) + b^* u''(c^*)\right).$$

Because $D^* > 0$, we conclude that $\frac{\partial \ell^*}{\partial r_b} = 0$ if $A^*_\ell = 0$. If $A^*_\ell > 0$, then we conclude that:

$$\frac{\partial \ell^*}{\partial r_b} \geq 0 \iff u'(c^*) + b^* u''(c^*) \leq 0 \iff b^* \geq \frac{u'(c^*)}{-u''(c^*)} = T_u(c^*).$$

(ii). To prove that consumption is decreasing in r_b, apply (EC.4) with $\eta = r_b$ to obtain:

$$D^*\frac{\partial c^*}{\partial r_b} = \bar{V}^*_{\ell c}\bar{V}^*_{\ell r_b} - \bar{V}^*_{\ell\ell}\bar{V}^*_{c r_b} = r_b b^*(A^*_\ell)^2 + \left(M^* + r_b b^* C^*\right)\bar{V}^*_{\ell\ell}.$$

We will bound the first term. First note that (EC.3) implies that:

$$\bar{V}^*_{\ell\ell} = -d''(\ell^*) + \mathbb{E}\left[v''(\bar{Y}^*)(\Phi^*_\ell)^2 + v'(\bar{Y}^*)\Phi^*_{\ell\ell}\right] \leq \mathbb{E}\left[v''(\bar{Y}^*)(\Phi^*_\ell)^2\right],$$

where the inequality follows because $d'' > 0$, $v' > 0$, and $\Phi^*_{\ell\ell} \leq 0$. The Cauchy–Bunyakovsky–Schwarz inequality then implies that:

$$(A^*_\ell)^2 = \left(\mathbb{E}[-v''(\bar{Y}^*)\Phi^*_\ell]\right)^2 \leq \mathbb{E}[-v''(\bar{Y}^*)]\,\mathbb{E}[-v''(\bar{Y}^*)(\Phi^*_\ell)^2] \leq -C^*\bar{V}^*_{\ell\ell}.$$

Therefore, $D^*\frac{\partial c^*}{\partial r_b} \leq -r_b b^* C^* \bar{V}^*_{\ell\ell} + \left(M^* + r_b b^* C^*\right)\bar{V}^*_{\ell\ell} = M^*\bar{V}^*_{\ell\ell} < 0$. Because $D > 0$, we have $\frac{\partial c^*}{\partial r_b} < 0$.

(iii). The welfare derivative follows directly from the envelope theorem. Because $x - c^* < 0$ and $v' > 0$, we obtain:

$$\frac{\partial \bar{V}^*}{\partial r_b} = \bar{V}_{r_b}(\ell^*, c^*; r_b, x, \theta) = (x - c^*)\mathbb{E}[v'(\bar{Y}^*)] < 0. \quad \square$$

Proof of Corollary 1. In the borrowing regime, $\bar{V}$ is twice differentiable and $\bar{Y}(\ell, c; x, \theta, \xi) = \Phi(\ell, \theta, \xi) + r_b(x - c)$. Proposition 1 applied within the borrowing regime yields

$$\frac{\partial b^*(x)}{\partial x} = \frac{\partial c^*(x)}{\partial x} - 1 < 0,$$

and thus $b^*(x)$ is strictly decreasing in x. Since $c^*(x)$ is weakly increasing in x and T_u is weakly increasing in c, the composite $T_u(c^*(x))$ is weakly increasing in x. It follows that

$$\psi(x) = b^*(x) - T_u(c^*(x))$$

is strictly decreasing on $\mathcal{X}_b$. In particular, ψ has at most one zero.

To connect this with the effect of improved credit access, recall from Proposition 2's proof that

$$D^* \frac{\partial \ell^*}{\partial r_b} = A_\ell^*[-u''(c^*)]\left[b^* - T_u(c^*)\right],$$

where $A_\ell^* := \mathbb{E}[-v''(\bar{Y}^*)\Phi_\ell^*] \geq 0$. Showing the dependence on x and recalling $\psi(x)$, this becomes:

$$D^*(x)\frac{\partial \ell^*(x)}{\partial r_b} = A_\ell^*(x)[-u''(c^*(x))]\psi(x).$$

Since $D^*(x) > 0$ and $-u''(c^*(x)) > 0$, the sign of $\frac{\partial \ell^*(x)}{\partial r_b}$ equals the sign of $\psi(x)$ whenever $A_\ell^*(x) > 0$, and equals zero whenever $A_\ell^*(x) = 0$. Since ψ is strictly decreasing and changes sign at most once on $\mathcal{X}_b$, the threshold $\bar{x}$ exists and the stated monotone pattern follows. □

EC.2.4. Proofs for Results in §4.3 on Savings Access

Proof of Proposition 3. Fix an interior optimum in the saving regime. Then $s^* := x - c^* > 0$ and $\bar{Y}^* = \bar{Y}(\ell^*, c^*; x, \theta, \xi) = \Phi(\ell^*, \theta, \xi) + r_s s^*$.

For compactness, write

$$A_\ell^* := \mathbb{E}[-v''(\bar{Y}^*)\Phi_\ell^*] \geq 0, \qquad C^* := \mathbb{E}[-v''(\bar{Y}^*)] \geq 0, \qquad M^* := \mathbb{E}[v'(\bar{Y}^*)] > 0, \qquad \text{and}$$

$$H^* := -\bar{V}_{\ell\ell}^* > 0, \qquad D^* := \bar{V}_{\ell\ell}^*\bar{V}_{cc}^* - (\bar{V}_{\ell c}^*)^2 > 0,$$

where the inequalities follow from strict concavity. From (EC.3), evaluated with active rate $\rho = r_s$,

$$\bar{V}_{\ell c}^* = r_s A_\ell^*, \qquad \bar{V}_{\ell r_s}^* = -s^* A_\ell^*, \qquad \bar{V}_{c r_s}^* = -M^* + r_s s^* C^*, \qquad \bar{V}_{cc}^* = u''(c^*) - r_s^2 C^*.$$

(i). For child labor, (EC.4) gives

$$\begin{aligned}
D^* \frac{\partial \ell^*}{\partial r_s} &= \bar{V}_{\ell c}^*\bar{V}_{c r_s}^* - \bar{V}_{cc}^*\bar{V}_{\ell r_s}^* = r_s A_\ell^*(-M^* + r_s s^* C^*) - (u''(c^*) - r_s^2 C^*)(-s^* A_\ell^*) \\
&= -A_\ell^*\left[r_s M^* - s^* u''(c^*)\right] \\
&= -A_\ell^*\left[u'(c^*) - s^* u''(c^*)\right] \leq 0,
\end{aligned}$$

where the second equality follows from the consumption first-order condition, $u'(c^*) = r_s\mathbb{E}[v'(\bar{Y}^*)] = r_s M^*$, and the inequality follows because $A_\ell \geq 0, u' > 0, s^* > 0, u'' < 0$. Because $D^* > 0$, we conclude that $A_\ell^* = 0$ implies that labor remains unchanged with improved savings access, $\frac{\partial \ell^*}{\partial r_s} = 0$. In contrast, $A_\ell^* > 0$ implies a strict reduction in child labor, $\frac{\partial \ell^*}{\partial r_s} < 0$.

(ii). For consumption, (EC.4) gives

$$\begin{aligned}
D^* \frac{\partial c^*}{\partial r_s} &= \bar{V}_{\ell c}^*\bar{V}_{\ell r_s}^* - \bar{V}_{\ell\ell}^*\bar{V}_{c r_s}^* \\
&= -r_s s^*(A_\ell^*)^2 + H^*(-M^* + r_s s^* C^*).
\end{aligned}$$

Since $D^* > 0$ and $H^* > 0$,

$$\frac{\partial c^*}{\partial r_s} \le 0 \quad \Longleftrightarrow \quad H^*(r_s s^* C^* - M^*) \le r_s s^* (A_\ell^*)^2 \quad \Longleftrightarrow \quad r_s s^* \left(C^* - \frac{(A_\ell^*)^2}{H^*} \right) \le M^*.$$

Substituting the definitions of A_ℓ^*, C^*, M^*, and H^* gives

$$\frac{\partial c^*}{\partial r_s} \le 0 \quad \Longleftrightarrow \quad r_s s^* \left(\mathbb{E}[-v''(\bar{Y}^*)] - \frac{\left(\mathbb{E}[-v''(\bar{Y}^*)\Phi_\ell^*]\right)^2}{-\bar{V}_{\ell\ell}^*} \right) \le \mathbb{E}[v'(\bar{Y}^*)].$$

Replacing $\le$ by $<$ throughout gives the strict condition, and equality gives $\partial c^*/\partial r_s = 0$.

(iii). The welfare response follows from the envelope theorem:

$$\frac{\partial \bar{V}^*}{\partial r_s} = \bar{V}_{r_s}^* = (x - c^*)\mathbb{E}[v'(\bar{Y}^*)] = s^* \mathbb{E}[v'(\bar{Y}^*)] > 0,$$

because $s^* > 0$ and $v' > 0$. Hence household welfare strictly increases in r_s. □

EC.2.5. Proofs for Results in §4.4 on Production Support

Proof of Proposition 4. *We prove (i) and (ii) together.* Consider the financially neutral regime, $c^* = x$. Since the household remains financially neutral locally, $c^*(\theta) = x$, and therefore $\frac{\partial c^*}{\partial \theta} = 0$. In this regime, $\bar{Y}^* = \Phi^* := \Phi(\ell^*, \theta, \xi)$, and the labor first-order condition is (EC.7), i.e., $0 = -d'(\ell^*) + \mathbb{E}\left[v'(\Phi^*)\Phi_\ell^*\right]$. Differentiating this with respect to θ gives

$$\begin{aligned} 0 &= \left\{-d''(\ell^*) + \mathbb{E}\left[v''(\Phi^*)(\Phi_\ell^*)^2 + v'(\Phi^*)\Phi_{\ell\ell}^*\right]\right\} \frac{\partial \ell^*}{\partial \theta} + \mathbb{E}\left[v''(\Phi^*)\Phi_\ell^*\Phi_\theta^* + v'(\Phi^*)\Phi_{\ell\theta}^*\right] \\ &= -\Omega_\ell^* \frac{\partial \ell^*}{\partial \theta} + P_\theta^* - R_\theta^*, \end{aligned}$$

where the second equality follows from the definitions of R_θ^* and P_θ^* and recognizing that $C_\theta^* = 0$ when $c^* = x$, and where $\Omega_\ell^* := d''(\ell^*) - \mathbb{E}\left[v''(\Phi^*)(\Phi_\ell^*)^2 + v'(\Phi^*)\Phi_{\ell\ell}^*\right] > 0$ because $d'' > 0$. Thus,

$$\frac{\partial \ell^*}{\partial \theta} \le 0 \quad \Longleftrightarrow \quad R_\theta^* \ge P_\theta^* = P_\theta^* + C_\theta^*,$$

with strict inequality giving a strict decrease.

Lastly, consider the borrowing or saving regime, and let $\rho^* \in \{r_b, r_s\}$ denote the active rate. Recall from §EC.2.1 and Proposition 1 that locally, $\bar{Y}$ and $\bar{V}$ are twice differentiable in (ℓ, c). The interior first-order conditions are (EC.1a) and (EC.1b), and we have

$$D^* \frac{\partial \ell^*}{\partial \theta} = \bar{V}_{\ell c}^* \bar{V}_{c\theta}^* - \bar{V}_{cc}^* \bar{V}_{\ell\theta}^* \qquad \text{and} \qquad D^* \frac{\partial c^*}{\partial \theta} = \bar{V}_{\ell c}^* \bar{V}_{\ell\theta}^* - \bar{V}_{\ell\ell}^* \bar{V}_{c\theta}^*.$$

where $D^* > 0$. Using the expressions for cross derivatives from (EC.3) and recognizing the definitions of $R_\theta^*, P_\theta^*, C_\theta^*$ gives:

$$D^* \frac{\partial \ell^*}{\partial \theta} = (-\bar{V}_{cc}^*) \left[P_\theta^* + C_\theta^* - R_\theta^*\right] \qquad \text{and} \qquad D^* \frac{\partial c^*}{\partial \theta} = \bar{V}_{\ell c}^* (P_\theta^* - R_\theta^*) + (-\bar{V}_{\ell\ell}^*) \bar{V}_{c\theta}^*.$$

Since $-\bar{V}^*_{cc} > 0$ and $D^* > 0$, the claimed results follow.

(ii). To prove the welfare result, note that in any fixed financial regime, the envelope theorem gives:

$$\frac{\partial \bar{V}^*(\theta)}{\partial \theta} = \mathbb{E}\left[v'(\bar{Y}^*)\Phi^*_\theta\right].$$

Because $v' > 0$, $\Phi^*_\theta \geq 0$ a.s., and $\mathbb{P}(\Phi^*_\theta > 0) > 0$, this derivative is strictly positive. □

Proof of Corollary 2. Under the multiplicative specification, $\Phi(\ell,\theta,\xi) = \xi F(\ell,\theta)$, we have $\Phi_\ell = \xi F_\ell, \Phi_\theta = \xi F_\theta, \Phi_{\ell\theta} = \xi F_{\ell\theta}$. Fix x and evaluate all quantities at the corresponding optimum, with $F^* := F(\ell^*,\theta), q^* := x - c^*, U^* := -u''(c^*), \widehat{\rho}^* := \widehat{\rho}(c^*; x)$. Also, define for $j = 0, 1, 2$,

$$A^*_j := \mathbb{E}\left[-v''(F^*\xi + \widehat{\rho}^* q^*)\xi^j\right], \qquad M^*_1 := \mathbb{E}[v'(F^*\xi + \widehat{\rho}^* q^*)\xi],$$

By Proposition 4,

$$\frac{\partial \ell^*}{\partial \theta} \leq 0 \quad \Longleftrightarrow \quad R^*_\theta \geq P^*_\theta + C^*_\theta.$$

Using the multiplicative derivatives above,

$$R^*_\theta = F^*_\ell F^*_\theta A^*_2, \qquad P^*_\theta = F^*_{\ell\theta} M^*_1, \qquad C^*_\theta = \frac{(\widehat{\rho}^*)^2 F^*_\ell F^*_\theta (A^*_1)^2}{U^* + (\widehat{\rho}^*)^2 A^*_0}.$$

The expression for C^*_θ also holds for $c^* = x$ because $\hat{\rho} = 0$ by definition in that case. Hence:

$$R^*_\theta - P^*_\theta - C^*_\theta = F^*_\ell F^*_\theta \left[A^*_2 - \frac{(\widehat{\rho}^*)^2 (A^*_1)^2}{U^* + (\widehat{\rho}^*)^2 A^*_0}\right] - F^*_{\ell\theta} M^*_1.$$

Since $F^*_\ell > 0$, $F^*_\theta > 0$, $F^* > 0$, and $M^*_1 > 0$, the inequality $R^*_\theta \geq P^*_\theta + C^*_\theta$ is equivalent to

$$\frac{F^* F^*_{\ell\theta}}{F^*_\ell F^*_\theta} \leq \frac{F^*\left[A^*_2 - \dfrac{(\widehat{\rho}^*)^2 (A^*_1)^2}{U^* + (\widehat{\rho}^*)^2 A^*_0}\right]}{M^*_1}.$$

The left-hand side is $K(\ell^*,\theta)$, and the right-hand side is $B^*(x) = B_{\widehat{\rho}^*}(F^*, q^*, U^*)$. Therefore

$$\frac{\partial \ell^*}{\partial \theta} \leq 0 \quad \Longleftrightarrow \quad K(\ell^*,\theta) - B^*(x) \leq 0. \tag{EC.8}$$

The strict statement follows from the same equivalence.

Proof of (a). If $F_{\ell\theta} \leq 0$, then $K(\ell^*,\theta) \leq 0$. But we claim that $B^*(x) \geq 0$. To see this, note that the Cauchy–Bunyakovsky–Schwarz inequality yields $(A^*_1)^2 \leq A^*_0 A^*_2$, and therefore,

$$\frac{(\widehat{\rho}^*)^2 (A^*_1)^2}{U^* + (\widehat{\rho}^*)^2 A^*_0} \leq \frac{(\widehat{\rho}^*)^2 A^*_0 A^*_2}{U^* + (\widehat{\rho}^*)^2 A^*_0} \leq A^*_2,$$

because $U^* > 0$. Hence the bracket in $B_{\widehat{\rho}^*}$ is positive, and $B^*(x) \geq 0$. Therefore, $K(\ell^*,\theta) \leq 0 \leq B^*(x)$, and so production support weakly reduces child labor for every x. If $F_{\ell\theta} < 0$ at the relevant optimum, then $K(\ell^*,\theta) < 0 \leq B^*(x)$, so the reduction is strict.

Proof of (b)-(i). With $\Delta_\theta(x) := K(\ell^*(x), \theta) - B^*(x)$, the inequality (EC.8) implies that:

$$\frac{\partial \ell^*(x)}{\partial \theta} \leq 0 \quad \Longleftrightarrow \quad \Delta_\theta(x) \leq 0.$$

(b)-(ii-iii). Consider first the borrowing or saving regimes, so $x < x_1$ or $x > x_2$. Proposition 1 implies that $\ell^*(x)$ is decreasing in x, $c^*(x)$ is increasing in x, and $q^*(x) = x - c^*(x)$ is strictly increasing in x. Since $K_\ell \leq 0$, the composite $K(\ell^*(x), \theta)$ is increasing in x. Since $F_\ell > 0$, the term $F^*(x) := F(\ell^*(x), \theta)$ is decreasing in x. Since u'' is increasing in c, then $U^*(x) := -u''(c^*(x))$ is decreasing in x. By assumption, $B_{\widehat{\rho}}(F, q, U)$ is increasing in F, U and strictly decreasing in q. Therefore, along the borrowing and saving regimes, $B^*(x) = B_{\widehat{\rho}^*}(F^*(x), q^*(x), U^*(x))$ is strictly decreasing in x and thus $\Delta_\theta(x) = K(\ell^*(x), \theta) - B^*(x)$ is strictly increasing in x.

Now consider the financially neutral regime, $x_1 < x < x_2$. In this region, $c^*(x) = x, q^*(x) = 0, \ell^*(x) = \ell^0$, where ℓ^0 is the financially neutral labor choice. Hence

$$K(\ell^*(x), \theta) = K(\ell^0, \theta), \qquad B^*(x) = B_0(F(\ell^0, \theta))$$

are both constant in x. Thus $\Delta_\theta(x)$ is constant on (x_1, x_2).

Next we compare one-sided limits at the regime thresholds. At x_1, the one-sided optimal decisions converge to the financially neutral optimum, so K has the same one-sided limit on both sides. The only change in $B^*(x)$ is that the consumption-feedback term C_θ^* disappears when entering the financially neutral region. Let $F^0 := F(\ell^0, \theta)$; then, the relevant limits are:

$$\lim_{x \downarrow x_1} B^*(x) = B_0(F^0) = \frac{F^0 A_2^0}{M_1^0}, \quad \text{where} \quad A_j^0 := \mathbb{E}[-v''(F^0\xi)\xi^j], \qquad M_1^0 := \mathbb{E}[v'(F^0\xi)\xi$$

$$\lim_{x \uparrow x_1} B^*(x) = B_{b,1} = F^0 \left[A_2^0 - \frac{r_b^2 (A_1^0)^2}{-u''(x_1) + r_b^2 A_0^0} \right] / M_1^0$$

Therefore, $B_0(F^0) \geq B_{b,1}$. Since K is unchanged at the boundary, we have $\Delta(x_1^+) \leq \Delta(x_1^-)$ and thus Δ has a downward jump at x_1.

Similarly, at x_2, the saving-side threshold reintroduces the consumption-feedback term. Hence $B_0(F^0) \geq B_{s,2}$, where $B_{s,2} = \lim_{x \downarrow x_2} B^*(x)$. Therefore $\Delta_\theta(x_2^+) \geq \Delta_\theta(x_2^-)$, so Δ_θ jumps upward at x_2.

The threshold representation now follows from the monotonicity of Δ_θ. Since Δ_θ is increasing on the borrowing region, it can cross zero at most once there. Since it is constant in the financially neutral region, it cannot cross zero inside (x_1, x_2). Since it is increasing in the saving region, it can cross zero at most once there. The downward jump at x_1 and the upward jump at x_2 account for the remaining possible sign switches.

More explicitly, define $x_1' := \sup\{x < x_1 : \Delta_\theta(x) \leq 0\}$, with the conventions $x_1' = -\infty$ if the set is empty and $x_1' = x_1$ if $\Delta_\theta(x) \leq 0$ for all $x < x_1$. Let Δ_θ^0 denote the common value of $\Delta_\theta(x)$ on (x_1, x_2). If $\Delta_\theta^0 \leq 0$, set $x_2' := x_1, \qquad x_3' := \sup\{x > x_2 : \Delta_\theta(x) \leq 0\}$, with the conventions $x_3' = x_2$ if the set is empty and $x_3' = +\infty$ if

$\Delta_\theta(x) \le 0 \, \forall x > x_2$. If $\Delta_\theta^0 > 0$, set $x_2' := x_3' := x_2$. With these conventions, $x_1' \in [-\infty, x_1]$, $x_2' \in \{x_1, x_2\}$, $x_3' \in [x_2, +\infty]$, and the sign pattern in the corollary follows from part (b)-(i).

Finally, suppose all borrowing households reduce child labor. Then $\Delta_\theta(x) \le 0$ for all $x < x_1$, so $x_1' = x_1$. Since Δ_θ jumps downward at x_1, $\Delta_\theta^0 = \Delta_\theta(x_1^+) \le \Delta_\theta(x_1^-) \le 0$, so the financially neutral region is also child-labor reducing, and $x_2' = x_1$.

Conversely, suppose the financially neutral region increases child labor. Then $\Delta_\theta^0 > 0$. Since Δ_θ jumps upward at x_2, it follows that $\Delta_\theta(x_2^+) \ge \Delta_\theta^0 > 0$. Because Δ_θ is increasing in the saving regime, $\Delta_\theta(x) > 0$ for all $x > x_2$. Hence all saving households increase child labor, and $x_3' = x_2$. This proves the corollary. □

EC.3. Extensions

EC.3.1. Cash Transfer Results

We discuss two extensions for the results in §4.1. The first concerns a household faced with a minimum consumption level, $c \ge c_s + \alpha x$, where c_s is a fixed subsistence consumption level and $\alpha \in [0,1)$ is a fraction of the household's cash devoted to consumption. Our next result shows that in the presence of a binding minimum consumption constraint, a cash transfer would strictly increase the household's welfare, would increase its immediate consumption (strictly if $\alpha > 0$), and would lower its use of child labor (strictly under a condition analogous to that in Proposition 1.)

Proposition EC.1 **(Minimum consumption constraint)**. *Suppose the household faces the consumption constraint $c \ge c_s + \alpha x$, where $\alpha \in [0,1)$. Consider an open interval of cash positions x on which this constraint binds, child labor is interior, and the active financial regime is fixed with relevant rate $\rho = r_b$ if $x < c_s + \alpha x$ and with $\rho = r_s$ if $x > c_s + \alpha x$. Then,*

(i) Child labor $\ell^*(x)$ weakly decreases in x. It is strictly decreasing if and only if $\mathbb{E}\left[v''(\bar{Y}^*)\Phi_\ell(\ell^*(x),\theta,\xi)\right] < 0$, where $\bar{Y}^* = \Phi(\ell^*(x),\theta,\xi) + \rho q^*(x)$).

(ii) Current consumption $c^*(x)$ weakly increases in x (and strictly increases if and only if $\alpha > 0$)

(iii) Household's welfare $\bar{V}^*(x)$ strictly increases in x.

Proof of Proposition EC.1. Because the constraint is binding, we have

$$c^*(x) = c_s + \alpha x, \qquad q^*(x) = x - c^*(x) = (1-\alpha)x - c_s.$$

This immediately gives

$$\frac{\partial c^*(x)}{\partial x} = \alpha, \qquad \frac{\partial q^*(x)}{\partial x} = 1 - \alpha.$$

Fix a borrowing or saving region, so the active rate ρ is locally constant and recall the notation

$$\bar{Y}^* = \Phi(\ell^*(x),\theta,\xi) + \rho q^*(x).$$

Because child labor is interior, the labor first-order condition is

$$-d'(\ell^*(x)) + \mathbb{E}\left[v'(\bar{Y}^*)\Phi_\ell^*\right] = 0.$$

Differentiating this condition with respect to x gives

$$0 = \left\{ -d''(\ell^*(x)) + \mathbb{E}\Big[v''(\bar{Y}^*)\big(\Phi_\ell^*\big)^2 + v'(\bar{Y}^*)\Phi_{\ell\ell}^*\Big]\right\} \frac{\partial \ell^*(x)}{\partial x} + \rho(1-\alpha)\mathbb{E}\big[v''(\bar{Y}^*)\Phi_\ell^*\big] .$$

Define

$$\Omega_\ell^* := d''(\ell^*(x)) - \mathbb{E}\Big[v''(\bar{Y}^*)\big(\Phi_\ell^*\big)^2 + v'(\bar{Y}^*)\Phi_{\ell\ell}^*\Big] .$$

Then $\Omega_\ell^* > 0$ because $d'' > 0$, $v'' \le 0$, $v' > 0$, and $\Phi_{\ell\ell} \le 0$. Hence the differentiated first-order condition can be written as

$$0 = -\Omega_\ell^* \frac{\partial \ell^*(x)}{\partial x} + \rho(1-\alpha)\mathbb{E}\big[v''(\bar{Y}^*)\Phi_\ell^*\big] ,$$

or equivalently

$$\frac{\partial \ell^*(x)}{\partial x} = \frac{\rho(1-\alpha)}{\Omega_\ell^*}\mathbb{E}\big[v''(\bar{Y}^*)\Phi_\ell^*\big] .$$

Since $\rho > 0$, $1-\alpha > 0$, $\Omega_\ell^* > 0$, $v'' \le 0$, and $\Phi_\ell^* \ge 0$, it follows that $\frac{\partial \ell^*(x)}{\partial x} \le 0$. Moreover, the inequality is strict if and only if $\mathbb{E}\big[v''(\bar{Y}^*)\Phi_\ell^*\big] < 0$, equivalently $\mathbb{E}[-v''(\bar{Y}^*)\Phi_\ell^*] > 0$.

Finally, welfare strictly increases by differentiating $\bar{V}$ along the binding constraint:

$$\begin{aligned}\frac{\partial \bar{V}^*(x)}{\partial x} &= u'(c^*(x))\frac{\partial c^*(x)}{\partial x} + \mathbb{E}[v'(\bar{Y}^*)]\rho\frac{\partial q^*(x)}{\partial x} \\ &= \alpha u'(c^*(x)) + \rho(1-\alpha)\mathbb{E}[v'(\bar{Y}^*)] > 0,\end{aligned}$$

where the second equality follows by substituting $\partial c^*/\partial x = \alpha$ and $\partial q^*/\partial x = 1-\alpha$, and the last inequality follows from $u' > 0$, $v' > 0$, $\rho > 0$, and $\alpha \in [0,1)$. □

The next extension concerns a household faced with a constraint on its use of child labor, $\ell \le \bar{\ell}$. We show that if this constraint is binding, a cash transfer leaves child labor unchanged ($\ell = \ell^*$), but it lowers the shadow price of the child labor constraint. It also increases current consumption (strictly if $\alpha > 0$), and strictly increases the household's welfare.

Proposition EC.2. *Suppose the household faces the constraint $\ell \le \bar{\ell}$, and consider a range of cash position values x on which $\ell^*(x) = \bar{\ell}$ and the active financial regime is fixed with relevant rate $\rho \in \{r_b, r_s\}$. Then,*

(i) Child labor $\ell^*(x)$ is unchanged in x, but the dual variable (i.e., the shadow price) of the child labor constraint decreases in x.

(ii) Current consumption $c^*(x)$ increases in x.

(iii) Household welfare $\bar{V}^*(x)$ strictly increases in x.

Proof of Proposition EC.2. That $\bar{V}(x)$ is strictly increasing, $c^*(x)$ is increasing, and $q^*(x)$ is increasing follows from identical arguments as those in the proof of Proposition 1. The arguments in that proof can also be invoked to show that the same three financial regimes can arise here as well; for simplicity, we restrict the arguments here to a fixed financial regime denoted by ρ. The optimal dual variable for the child-labor constraint can then be derived as:

$$\mu_\ell(x) := -d'(\bar{\ell}) + \mathbb{E}\left[v'(\bar{Y}(\bar{\ell}, c^*(x); x, \theta, \xi))\Phi_\ell(\bar{\ell}, \theta, \xi)\right]$$

Differentiating this with respect to x yields:

$$\frac{\partial \mu_\ell(x)}{\partial x} = \bar{V}^*_{\ell c}\left(\frac{\partial c^*(x)}{\partial x} - 1\right) \le 0. \quad \square$$

EC.3.2. Endogenous purchased inputs and mixed cash interventions

The main text treats cash transfers as pure-liquidity shifts: the intervention raises x while leaving the production technology fixed. This benchmark is useful because it isolates the constraint-relief channel. Some households, however, may allocate their liquidity among consumption, savings, and purchases of production-improving inputs such as fertilizer or hired labor. This extension shows that the main taxonomy is robust to that possibility. A cash intervention can become a mixed instrument: it can relax liquidity and also induce production expansion. The same logic applies to credit programs that finance inputs, although the formal diagnostic below is written for a marginal increase in liquidity.

Let $a \ge 0$ denote a purchased productive input, and let $p > 0$ be its unit price. The input is chosen at the start of the season and must be financed out of the household's liquidity, savings, or borrowing. Production is now $\Phi(\ell, a, \theta, \xi)$.

The household solves

$$\begin{aligned} \max_{\ell,c,a,s,b} \quad & u(c) - d(\ell) + \mathbb{E}[v(\Phi(\ell, a, \theta, \xi) + r_s s - r_b b)] \\ \text{s.t.} \quad & a \ge 0, \quad s \ge 0, \quad b \ge 0, \\ & c + pa + s \le x + b. \end{aligned} \tag{EC.9}$$

Assume that Φ is increasing and concave in ℓ for each (a, θ, ξ), twice differentiable in (ℓ, a) on the relevant domain, and that all expectations and derivatives below are well-defined.

Lemma EC.2 **(No simultaneous borrowing and saving with purchased inputs)**. *For any optimum of* (EC.9)*,*

$$b^* = (c^* + pa^* - x)^+, \qquad s^* = (x - c^* - pa^*)^+.$$

Proof Fix any feasible (ℓ, c, a) and any feasible financial choice (s, b). Let

$$m := x - c - pa, \qquad F(s, b) := r_s s - r_b b.$$

The budget constraint in (EC.9) implies

$$c + pa + s \le x + b \qquad \Longleftrightarrow \qquad s - b \le x - c - pa = m.$$

The rest of the argument is identical to Lemma 1. If $m \ge 0$, then for any feasible (s, b),

$$F(s,b) = r_s(s-b) - (r_b - r_s)b \le r_s m,$$

with equality only when $b = 0$ and $s = m = x - c - pa$. If $m < 0$, then

$$F(s,b) = r_b(s-b) - (r_b - r_s)s \le r_b m,$$

with equality only when $s = 0$ and $b = -m = c + pa - x$. As $v : \mathbb{R} \to \mathbb{R}$ and v is strictly increasing, any other feasible financial choice is dominated. Hence

$$b^* = (c^* + pa^* - x)^+, \qquad s^* = (x - c^* - pa^*)^+. \qquad \square$$

By Lemma EC.2, the household again has an active financial regime. Fix such a regime and let $r \in \{r_b, r_s\}$ denote its gross rate. Define the net pre-harvest financial position after input purchases by $m^a := x - c - pa$, and define terminal resources by

$$\widetilde{Y}_r(\ell, c, a, \theta, \xi) := \Phi(\ell, a, \theta, \xi) + r(x - c - pa). \tag{EC.10}$$

The reduced objective is

$$\widetilde{V}(\ell, c, a; r, x, \theta, p) = u(c) - d(\ell) + \mathbb{E}\left[v\left(\widetilde{Y}_r(\ell, c, a, \theta, \xi)\right)\right]. \tag{EC.11}$$

At an interior optimum with $a^* > 0$, the first-order conditions are

$$d'(\ell^*) = \mathbb{E}\left[v'(\widetilde{Y}_r^*)\Phi_\ell^*\right], \tag{EC.12}$$

$$u'(c^*) = r\mathbb{E}\left[v'(\widetilde{Y}_r^*)\right], \tag{EC.13}$$

$$\mathbb{E}\left[v'(\widetilde{Y}_r^*)\left(\Phi_a^* - rp\right)\right] = 0. \tag{EC.14}$$

Here

$$\widetilde{Y}_r^* := \widetilde{Y}_r(\ell^*, c^*, a^*, \theta, \xi), \qquad \text{and}$$

$$\Phi_\ell^* := \Phi_\ell(\ell^*, a^*, \theta, \xi), \qquad \Phi_a^* := \Phi_a(\ell^*, a^*, \theta, \xi), \qquad \Phi_{\ell a}^* := \Phi_{\ell a}(\ell^*, a^*, \theta, \xi).$$

The input first-order condition has a simple interpretation. The household buys the input until the continuation-value-weighted marginal product of the input equals its financed cost rp. If the household is borrowing, $r = r_b$, so the input is financed at the borrowing cost. If the household is saving, $r = r_s$, so the input is financed by reducing savings and the relevant cost is the opportunity cost of foregone saving.

The next result gives the exact local diagnostic for a cash transfer when input purchases are endogenous.

Proposition EC.3 (**Cash transfers with endogenous input purchases**). *Consider an interior, locally stable optimum of* (EC.11)*, away from the financial-regime kink and with $a^* > 0$. Let $(\ell^*(x), c^*(x), a^*(x))$ denote a differentiable local optimum branch for marginal changes in x that keep the household in the same active financial regime. Define $m^{a,*}(x) := x - c^*(x) - pa^*(x)$. At the baseline optimum, define $A_\ell^{a,*} := \mathbb{E}\Big[-v''(\widetilde{Y}_r^*)\Phi_\ell^*\Big] \geq 0$, $C^{a,*} := \mathbb{E}[-v''(\widetilde{Y}_r^*)] \geq 0$, and $\mathcal{B}_a^* = \mathcal{P}_a^* - \mathcal{I}_a^*$, where*

$$\mathcal{P}_a^* := \mathbb{E}\Big[v'(\widetilde{Y}_r^*)\Phi_{\ell a}^*\Big], \qquad \mathcal{I}_a^* := \mathbb{E}\left[\frac{v'(\widetilde{Y}_r^*)}{T_v(\widetilde{Y}_r^*)}\Phi_\ell^*\Phi_a^*\right].$$

Finally, let $\Omega_\ell^* := d''(\ell^*) - \mathbb{E}\Big[v''(\widetilde{Y}_r^*)(\Phi_\ell^*)^2 + v'(\widetilde{Y}_r^*)\Phi_{\ell\ell}^*\Big] > 0$. Then,

(i) Child labor use increases with x if and only if $rA_\ell^{a,*}\frac{\partial m^{a,*}}{\partial x}$ is greater than $\mathcal{B}_a^*\frac{\partial a^*}{\partial x}$,

$$\frac{\partial \ell^*}{\partial x} \leq 0 \quad \Longleftrightarrow \quad rA_\ell^{a,*}\frac{\partial m^{a,*}}{\partial x} \geq \mathcal{B}_a^*\frac{\partial a^*}{\partial x}.$$

(ii) The effect of an increase in x on consumption is characterized by:

$$\frac{\partial c^*}{\partial x} \geq 0 \quad \Longleftrightarrow \quad r\left(C^{a,*} - \frac{(A_\ell^{a,*})^2}{\Omega_\ell^*}\frac{\partial m^{a,*}}{\partial x}\right) + \left(\frac{A_\ell^{a,*}\mathcal{B}_a^*}{\Omega_\ell^*} + A_a^{a,*}\right)\frac{\partial a^*}{\partial x} \geq 0.$$

(iii) Household welfare $\widetilde{V}^*$ strictly increases in x.

Proof *(i)*. To derive the comparative static for ℓ^*, define the labor first-order condition by

$$F(\ell, c, a, x) := -d'(\ell) + \mathbb{E}\Big[v'(\widetilde{Y}_r(\ell, c, a, \theta, \xi))\Phi_\ell(\ell, a, \theta, \xi)\Big] = 0.$$

Along the differentiable local optimum branch, $F(\ell^*(x), c^*(x), a^*(x), x) = 0$.

Differentiate this identity with respect to x. Since $\widetilde{Y}_r = \Phi(\ell, a, \theta, \xi) + r(x - c - pa)$, the total derivative of $\widetilde{Y}_r$ along the branch is

$$\frac{\partial \widetilde{Y}_r^*}{\partial x} = \Phi_\ell^*\frac{\partial \ell^*}{\partial x} + \Phi_a^*\frac{\partial a^*}{\partial x} + r\left(1 - \frac{\partial c^*}{\partial x} - p\frac{\partial a^*}{\partial x}\right).$$

Using

$$\frac{\partial m^{a,*}}{\partial x} = 1 - \frac{\partial c^*}{\partial x} - p\frac{\partial a^*}{\partial x},$$

we obtain

$$\frac{\partial \widetilde{Y}_r^*}{\partial x} = \Phi_\ell^*\frac{\partial \ell^*}{\partial x} + \Phi_a^*\frac{\partial a^*}{\partial x} + r\frac{\partial m^{a,*}}{\partial x}.$$

Differentiating the labor first-order condition gives

$$\begin{aligned} 0 = & -d''(\ell^*)\frac{\partial \ell^*}{\partial x} + \mathbb{E}\left[v''(\widetilde{Y}_r^*)\frac{\partial \widetilde{Y}_r^*}{\partial x}\Phi_\ell^* + v'(\widetilde{Y}_r^*)\left(\Phi_{\ell\ell}^*\frac{\partial \ell^*}{\partial x} + \Phi_{\ell a}^*\frac{\partial a^*}{\partial x}\right)\right] \\ = & \Big[-d''(\ell^*) + \mathbb{E}\Big[v''(\widetilde{Y}_r^*)(\Phi_\ell^*)^2 + v'(\widetilde{Y}_r^*)\Phi_{\ell\ell}^*\Big]\Big]\frac{\partial \ell^*}{\partial x} \\ & + r\mathbb{E}\Big[v''(\widetilde{Y}_r^*)\Phi_\ell^*\Big]\frac{\partial m^{a,*}}{\partial x} + \mathbb{E}\Big[v''(\widetilde{Y}_r^*)\Phi_\ell^*\Phi_a^* + v'(\widetilde{Y}_r^*)\Phi_{\ell a}^*\Big]\frac{\partial a^*}{\partial x}. \end{aligned}$$

By definition, the first bracket is $-\Omega_\ell^*$, the second expectation equals $-A_\ell^{a,*}$, and the third expectation equals $\mathcal{B}_a^*$. Therefore

$$0 = -\Omega_\ell^* \frac{\partial \ell^*}{\partial x} - rA_\ell^{a,*} \frac{\partial m^{a,*}}{\partial x} + \mathcal{B}_a^* \frac{\partial a^*}{\partial x}.$$

Rearranging gives

$$\Omega_\ell^* \frac{\partial \ell^*}{\partial x} = -rA_\ell^{a,*} \frac{\partial m^{a,*}}{\partial x} + \mathcal{B}_a^* \frac{\partial a^*}{\partial x}.$$

Since $\Omega_\ell^* > 0$, its follows immediately that

$$\frac{\partial \ell^*}{\partial x} \le 0 \quad \iff \quad rA_\ell^{a,*} \frac{\partial m^{a,*}}{\partial x} \ge \mathcal{B}_a^* \frac{\partial a^*}{\partial x}.$$

Finally, using $v''(y) = -v'(y)/T_v(y)$, we can write

$$\mathcal{B}_a^* = \mathbb{E}\left[v'(\widetilde{Y}_r^*)\Phi_{\ell a}^*\right] - \mathbb{E}\left[\frac{v'(\widetilde{Y}_r^*)}{T_v(\widetilde{Y}_r^*)}\Phi_\ell^*\Phi_a^*\right] = \mathcal{P}_a^* - \mathcal{I}_a^*.$$

(ii). Moving on to c^*. For compactness, define

$$C^{a,*} := \mathbb{E}[-v''(\widetilde{Y}_r^*)] \ge 0, \qquad A_a^{a,*} := \mathbb{E}[-v''(\widetilde{Y}_r^*)(\Phi_a^* - rp)].$$

Differentiate the consumption first-order condition $u'(c^*) = r\mathbb{E}[v'(\widetilde{Y}_r^*)]$ along the branch, using the expression for $\frac{\partial \widetilde{Y}_r^*}{\partial x}$ from (i):

$$u''(c^*)\frac{\partial c^*}{\partial x} = r\mathbb{E}\left[v''(\widetilde{Y}_r^*)\left(\Phi_\ell^* \frac{\partial \ell^*}{\partial x} + \Phi_a^* \frac{\partial a^*}{\partial x} + r\frac{\partial m^{a,*}}{\partial x}\right)\right].$$

Write $\Phi_a^* = (\Phi_a^* - rp) + rp$ and use $\mathbb{E}[v''(\widetilde{Y}_r^*)] = -C^{a,*}$ and $\mathbb{E}[v''(\widetilde{Y}_r^*)(\Phi_a^* - rp)] = -A_a^{a,*}$:

$$u''(c^*)\frac{\partial c^*}{\partial x} = -rA_\ell^{a,*}\frac{\partial \ell^*}{\partial x} - r\left(A_a^{a,*} + rpC^{a,*}\right)\frac{\partial a^*}{\partial x} - r^2 C^{a,*}\frac{\partial m^{a,*}}{\partial x}.$$

Substitute $\frac{\partial m^{a,*}}{\partial x} = 1 - \frac{\partial c^*}{\partial x} - p\frac{\partial a^*}{\partial x}$ into the last term and collect the $\frac{\partial c^*}{\partial x}$ terms on the left:

$$\left(u''(c^*) - r^2 C^{a,*}\right)\frac{\partial c^*}{\partial x} = -rA_\ell^{a,*}\frac{\partial \ell^*}{\partial x} - rA_a^{a,*}\frac{\partial a^*}{\partial x} - r^2 C^{a,*},$$

where the $rpC^{a,*}\frac{\partial a^*}{\partial x}$ contributions cancel exactly. Let $D_c^{a,*} := r^2 C^{a,*} - u''(c^*) > 0$; then

$$D_c^{a,*}\frac{\partial c^*}{\partial x} = rA_\ell^{a,*}\frac{\partial \ell^*}{\partial x} + rA_a^{a,*}\frac{\partial a^*}{\partial x} + r^2 C^{a,*}.$$

Substitute $\Omega_\ell^*\frac{\partial \ell^*}{\partial x} = \mathcal{B}_a^*\frac{\partial a^*}{\partial x} - rA_\ell^{a,*}\frac{\partial m^{a,*}}{\partial x}$ from (i) and rearrange:

$$D_c^{a,*}\frac{\partial c^*}{\partial x} = r^2 C^{a,*} - \frac{r^2 (A_\ell^{a,*})^2}{\Omega_\ell^*}\frac{\partial m^{a,*}}{\partial x} + r\left(\frac{A_\ell^{a,*}\mathcal{B}_a^*}{\Omega_\ell^*} + A_a^{a,*}\right)\frac{\partial a^*}{\partial x}.$$

Since $D_c^{a,*} > 0$,

$$\frac{\partial c^*}{\partial x} \ge 0 \quad \iff \quad r\left(C^{a,*} - \frac{(A_\ell^{a,*})^2}{\Omega_\ell^*}\frac{\partial m^{a,*}}{\partial x}\right) + \left(\frac{A_\ell^{a,*}\mathcal{B}_a^*}{\Omega_\ell^*} + A_a^{a,*}\right)\frac{\partial a^*}{\partial x} \ge 0.$$

Replacing $\ge$ by $>$ throughout gives the strict condition.

(iii). The welfare effect of a marginal increase in x remains positive by the envelope theorem:

$$\frac{\partial \widetilde{V}^*}{\partial x} = r\mathbb{E}[v'(\widetilde{Y}_r^*)] \ge 0. \quad \square$$

Proposition EC.3 shows that endogenous input purchases do not introduce new mechanisms; they recombine the forces already identified in Propositions 1 and 4.

The child-labor condition in (i) decomposes into two forces. The first is a resource-relief force

$$r_\ell^{A,*} \frac{\partial m^{a,*}}{\partial x},$$

where when the transfer improves the net pre-harvest financial position $m^{a,*} = x - c^* - pa^*$, the marginal value of harvest cashflow falls and child labor is pushed downward. This is structurally identical to the resource-relief force in Proposition 1, now computed net of the liquidity absorbed by input purchases. When $\partial a^*/\partial x = 0$, child labor decreases in x exactly as in Proposition 1.

The second is a net labor-productivity force $\mathcal{B}_a^* \frac{\partial a^*}{\partial x}$, which arises only when the transfer induces additional input purchases. The coefficient $\mathcal{B}_a^* = \mathcal{P}_a^* - \mathcal{I}_a^*$ is itself the balance of two forces already present in Proposition 4. First, $\mathcal{P}_a^* = \mathbb{E}\left[v'(\tilde{Y}_r^*)\Phi_{\ell a}^*\right]$ is the labor-productivity force of the induced input. Second, $\mathcal{I}_a^* = \mathbb{E}\left[v'(\tilde{Y}_r^*)\, T_v(\tilde{Y}_r^*)^{-1}\Phi_\ell^*\Phi_a^*\right]$ is the resource-relief force generated by the input's own harvest returns. Thus $\mathcal{B}_a^*$ is the direct analog of $\mathcal{P}_\theta^* - \mathcal{R}_\theta^*$ from Proposition 4, restricted to the input margin.

Two special cases confirms that no new mechanism is at work. First, if the purchased input substitutes for child labor ($\Phi_{\ell a}^* \leq 0$) and $\Phi_\ell^* \geq 0$ almost surely, then $\mathcal{P}_a^* \leq 0$ and $\mathcal{I}_a^* \geq 0$, so $\mathcal{B}_a^* \leq 0$: the net labor-productivity force is non-positive and child labor weakly falls regardless of $\partial a^*/\partial x$, recovering the unambiguous direction of Proposition 1. Second, if the input is strongly complementary to child labor ($\Phi_{\ell a}^* > 0$) and its labor-productivity force dominates its own resource-relief contribution ($\mathcal{B}_a^* > 0$), a sufficiently large induced input purchase can cause child labor to rise, replicating the backfire logic of Proposition 4.